SPECIAL RELATIVITY PROBLEMS

Here are the answers – for step-by-step solutions of the calculation questions, see the video solution links.

1. Synchronized clocks are placed at regular intervals along the x-axis of your rest frame, separated by 1 billion kilometers. When the clock next to you reads midnight, what time do you see on the 60th clock down the line. What time does that clock really read?

200000 seconds earlier, or $55.\overline{5}$ hrs, or 4:27 p.m.

2. The lifetime of a muon at rest is 2.0 μsec. After this time it spontaneously disintegrates. If the muon is accelerated in a particle accelerator to a speed of 0.95c, how long will it survive?

$\gamma = 3.2$, so 6.4 μsec

3. (a) What is γ, in the limit of slow velocities? 1

(b) How about in the limit of v approaching c? ∞

4. How fast, in terms of c, must you be going for the length of your spaceship to appear 1/2 its rest length?

87% c

How fast for it to appear 1/10 the rest length?

99.5% c

5. As some outlaws escape in their getaway car, which is moving at 3c/4 (relative to the ground), a cop shoots a bullet from her police car, traveling at only c/2 (also relative to the ground). The muzzle velocity (speed relative to the gun) of the bullet is c/3. According to Galileo, does the bullet reach its target? yes
According to Einstein? no

6. In the year 2066, Earth builds a 120m-long spaceship of (rest) mass 1×10^6 kg. The ship takes off at 0.6c towards the star Gliese 699, which is 6 lightyears from Earth.

(a) Calculate γ. 1.25
(b) What is the apparent mass of the moving spaceship? 1.25×10^6 kg
(c) What is the length of the spaceship as measured by the Earth? 96m
(d) What is the length of the spaceship as measured by an astronaut on the spaceship? 120m
(e) How much time does it take the ship to arrive at Gliese 699, according to the astronaut? 8 years
(f) How long does it take the ship to arrive, according to Mission Control back on Earth? 10 years

7. The star Alpha Centauri is 4 light years from the Earth.

(a) If a spaceship flies off at 0.87c towards this star, what is γ? $\gamma = 2.0$
(b) How long does it take to get there, according to an astronaut on the ship? 2.3 years
(c) How long does it take to get there, according to people on Earth? 4.6 years
(d) When the ship arrives the astronaut sends a radio signal back to Earth. How long after the ship left does the signal arrive (according to clocks on Earth)? 8.6 years

8. A car with rest mass m'=1000 kg is moving at a speed of 0.1c.

(a) Calculate the kinetic energy of the car using the Newtonian expression for KE. 4.5×10^{17} J
(b) Using Einstein's equation $E=mc^2$, calculate the mass equivalent of that KE. 5 kg
(c) What then is the total mass of the moving car, i.e. its mass plus the mass of its KE? 1005 kg
(d) Show that the answer to (c) is the same as gotten by calculating $m=\gamma m'$.

$\gamma = 1.005$ so $\gamma m' = 1005$ kg

9. Explain why it is fundamentally impossible for a spaceship (or an electron in a particle accelerator) to reach or exceed the speed of light.

The mass, and therefore energy, would have to be infinite.

10. Suppose you are traveling in space and pass a rectangular landing pad on a planet. Your spacecraft

has a speed of 0.85c relative to the planet and moves in a direction parallel to the length of the pad. While moving, you measure the length of the pad to be 1800m and the width to be 1500m. What are the dimensions of the pad according to the engineers and construction workers who built it?

3400m x 1500m

11. The total amount of energy consumed in the U.S. in 1988 is estimated to have been about 8.4×10^{19}J. One penny has a mass of 2.9 g. If we could convert mass completely to energy, how many pennies worth of energy did the U.S. consume in 1988? 320,000 pennies

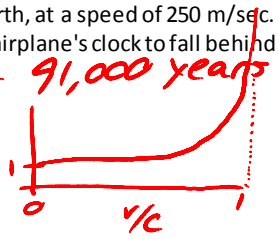
12. How fast does a spaceship need to go, so it is shrunk 1%?

.14c

13. An airplane is moving with respect to Earth, at a speed of 250 m/sec. How much time (as measured by clocks on the ground) will it take for the airplane's clock to fall behind 1.0 second?

2.9×10^{12} secs or 91,000 years

14. Sketch a graph of γ as a function of v.



15. (a) Calculate, in meters, how long a light-year is. 9.47×10^{15} m

(b) The diameter of the Earth is roughly 12800 km. How many Earth diameters is a light-year?

7.4×10^8

16. The USS Enterprise is in a battle with a Romulan starship. The Enterprise is moving at 0.9 c towards the Romulan ship, and fires a torpedo with a speed of 0.7c (relative to the Enterprise). How fast is the torpedo coming at the Romulan ship, according to the Romulans?

0.98c

17. What is the Correspondence Principle?

Any new mathematical law that "replaces" an old law must reproduce the predictions of the old law in those regimes where the old law is known to be valid. For example, the results of special relativity (time dilation, Lorentz contraction, and Einstein's velocity addition rule), in the limit of low velocities ($\gamma \approx 1$), match the "Newtonian" laws of physics (time flows at steady rate, things don't shrink when moving, and the Galilean velocity addition rule.)

18. A human being who lives about 72 years (a typical lifetime) will expend approximately 3×10^{11}J maintaining basic metabolism as well as a normal activity level. We get this energy by eating and digesting a lot of food during those 72 years. If we had stomachs that were able to convert food directly and completely into energy, how many kilograms of food would an average person have to consume in their lifetime?

3×10^{-6} kg

19. (a) The slowing of time in a moving frame is called... time dilation

(b) The shrinking of a moving object is called... Lorentz contraction