Heliosynchronous Orbit

The Sun rotates once every 25 days at its equator. (Because the Sun is not solid, different latitudes can and do rotate with somewhat different periods, but we're ignoring that here.)

(a) What is the radius, in AU, of a "heliosynchronous" orbit?
(b) Sketch a diagram of the Sun and Mercury, to scale, indicating where the heliosynchronous distance is.

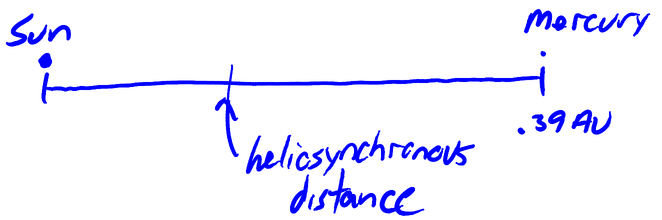
A few facts you need: the Sun's mass is $2.0\text{x}10^{30}$ kg, 1 AU is 150 million km, the radius of Mercury's orbit is 0.39 AU, and the radius of the Sun itself is 700,000 km. → 0.0047 AU

$$T^2 = \frac{4\pi^2 r^3}{G M} \quad \therefore \quad r^3 = \frac{T^2 M G}{4\pi^2}$$

$$T = 25\,\text{days}\left(\frac{86400\,\text{sec}}{1\,\text{day}}\right) = 2{,}160{,}000\,\text{sec} = 2.16\times10^{6}\,\text{sec}$$

$$r^3 = \frac{(2.16E6)^2 \cdot 2.0E30 \cdot 6.672E{-11}}{4\pi^2} = 1.58\times10^{31}$$

$$r = \sqrt[3]{1.58E31} = 2.5\times10^{10}\,\text{m} = 2.5\times10^{7}\,\text{km} = .17\,\text{AU} = 36\ \text{Sun radii}$$



By the way, you can also do this problem using the alternate units of AU, solar masses, and years instead of meters, kilograms and seconds. As discussed in the reading, this simplifies the math somewhat.

$$T^2 = \frac{r^3}{m} \quad \therefore \quad r^3 = mT^2$$

$$25\,\text{days}\left(\frac{1\,\text{yr}}{365\,\text{days}}\right) = .0685\,\text{yrs}$$

$$r^3 = m\cdot T^2 = 1\cdot 0.0685^2 = .00469$$

$$r = \sqrt[3]{.00469} = .17\,\text{AU}$$