

Instructions

You must sketch correct pictures and vectors, you must show all calculations, and you must explain all answers for full credit. Neatness and organization is required. Points will be taken off for sloppy work.

Other Equations

$$\Delta V = - \int_i^f \vec{E} \cdot d\vec{l} \text{ along any path from point } i \text{ to point } f \quad \vec{E} = -\nabla V = - \left\langle \frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z} \right\rangle$$

$$\Delta V = -(E_x \Delta x + E_y \Delta y + E_z \Delta z) \text{ for constant electric field or small displacement}$$

$$\Delta U = q\Delta V$$

$$|\vec{B}| = \frac{\mu_0}{4\pi} \frac{LI}{r\sqrt{r^2 + (L/2)^2}} \text{ along } \perp \text{ bisector of a straight wire} \quad |\vec{B}| = \frac{\mu_0}{4\pi} \frac{2I}{r} \quad r \ll L$$

$$|\vec{B}| = \frac{\mu_0}{4\pi} \frac{2IA}{(r^2 + R^2)^{3/2}} \text{ along axis of current-carrying loop} \quad |\vec{B}| \approx \frac{\mu_0}{4\pi} \frac{2IA}{r^3} \text{ for loop with } r \gg R$$

$$|\vec{B}| = \frac{\mu_0}{4\pi} \frac{2\mu}{r^3} \text{ along axis of magnetic dipole} \quad \mu = IA \text{ for current-carrying loop}$$

$$\text{Ohm's Law: } \Delta V = IR$$

$$\vec{F}_{mag} = q\vec{v} \times \vec{B} \quad \vec{F}_{mag} = I\vec{L} \times \vec{B}$$

$$U_{mag} = -\vec{\mu} \cdot \vec{B} \quad \vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\text{motional emf: } \epsilon = vBL$$

$$\epsilon = -\frac{d\Phi_{mag}}{dt} \quad \epsilon = \oint \vec{E}_{NC} \cdot d\vec{l}$$

Physical Constants

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}$$

$$e = 1.6 \times 10^{-19} \text{C}$$

$$\frac{\mu_0}{4\pi} = 1 \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}$$

$$c = 3 \times 10^8 \text{ m/s}^2$$

$$\text{Avogadro's Number} = 6.02 \times 10^{23} \text{ atoms/mole}$$

Geometry

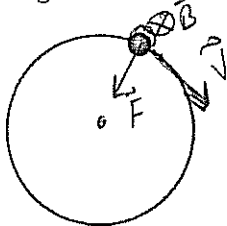
$$C = 2\pi R$$

$$\text{arclength} = R\Delta\theta$$

$$A = \pi R^2$$

Section 1. Multiple Choice

Questions 1-3: A *negatively* charged particle travels in uniform circular motion along a path of radius R in a region of uniform magnetic field, as shown below.



Define $+x$ to the right, $+y$ upward toward the top of the page, and $+z$ outward toward you, perpendicular to the page.

1. The direction of the magnetic field in this region is

- (a) $+x$
- (b) $-x$
- (c) $+y$
- (d) $-y$
- (e) $+z$
- (f) $-z$
- (g) None of the above because it is zero.

2. If another particle has a larger mass, but the same charge and speed, then the radius of its path will be

- (a) greater.
- (b) less.
- (c) the same.

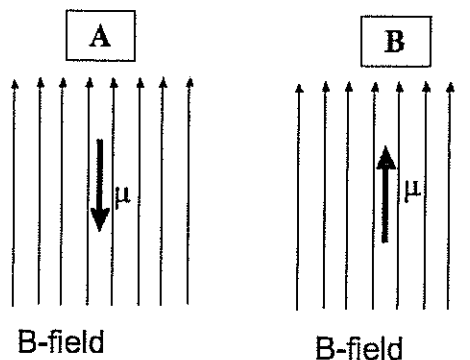
$$r \propto \frac{mv}{qB}$$

$$R = \frac{mv}{qB}$$

3. If another particle has a larger magnitude charge, but the same mass and speed, then the radius of its path will be

- (a) greater.
- (b) less.
- (c) the same.

Questions 4-6: An atomic magnetic dipole is in a uniform magnetic field and oriented opposite the field as shown in A. It then "flips" to the orientation shown in B. This is one part of the phenomenon that occurs in magnetic resonance imaging, for example.



4. Which energy state has a *higher* potential energy?

- (a) A
- (b) B
- (c) neither, because A and B have the same energy

5. As it changed from state A to state B, did the system emit a photon or absorb a photon?

- (a) absorb a photon
- (b) emit a photon
- (c) neither, because its energy did not change.

$$E_B < E_A \text{ so } E_f - E_i \text{ is } -$$

The system lost energy.

6. What is the magnitude of the change in the energy of the system and therefore the energy of the photon emitted or absorbed?

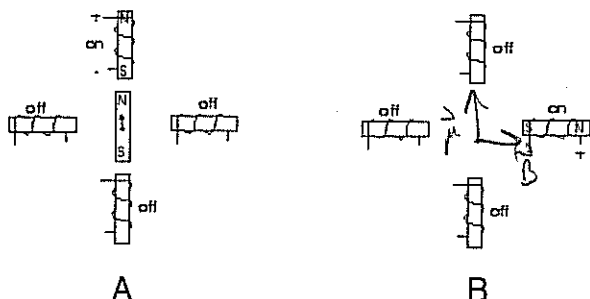
- (a) zero, because no photon is emitted or absorbed in this process
- (b) $\frac{1}{2}\mu B$
- (c) μB
- (d) $2\mu B$

$$U_A = -\mu B \cos(180^\circ) = \mu B$$

$$U_B = -\mu B \cos(0^\circ) = -\mu B$$

$$\Delta U = U_B - U_A = -2\mu B$$

Questions 7-8: The device below is designed to make a dipole magnet turn. In A, current flows through the top coil as shown in the diagram. In B, current flows through the coil on the right as shown. The dipole magnet in the middle of the device is not shown in diagram B.



7. As the device changes from A to B as shown in the diagram, will the dipole magnet in the middle rotate clockwise or counterclockwise?

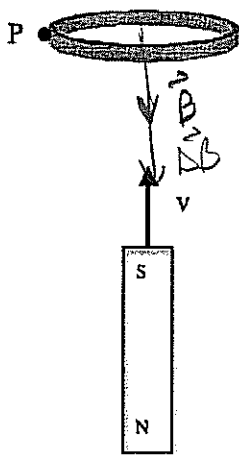
- (a) clockwise
- (b) counterclockwise
- (c) neither, because there is no torque on the dipole magnet

$\vec{\tau} = \vec{\mu} \times \vec{B}$
 $\vec{\tau}$ is in $-z$ dir.

8. In this device, electric current flowing through coils causes a magnetic dipole to rotate. What is this device called, in general?

- (a) the Zeta Magneteta
- (b) a freakin' awesome device
- (c) an electric motor
- (d) an electric generator

Questions 9-13: A magnet moves at constant speed toward a wire loop as shown below.



Define $+x$ to the right, $+y$ upward toward the top

of the page, and $+z$ outward toward you, perpendicular to the page.

9. At the center of the loop, what is the direction of the magnetic field?

- (a) $+x$
- (b) $-x$
- (c) $+y$
- (d) $-y$
- (e) $+z$
- (f) $-z$
- (g) None of the above because it is zero.

10. What is the direction of the change in the magnetic field at the center of the loop during some time interval Δt ?

- (a) $+x$
- (b) $-x$
- (c) $+y$
- (d) $-y$
- (e) $+z$
- (f) $-z$
- (g) None of the above because it is zero.

$\Delta \vec{B}$ is in same dir as \vec{B} since \vec{B} is increasing.

11. What is the direction of the electric field induced at location P inside the wire?

- (a) $+x$
- (b) $-x$
- (c) $+y$
- (d) $-y$
- (e) $+z$
- (f) $-z$
- (g) None of the above because it is zero.

thumb points in $+y$ dir.

12. Suppose that the emf induced around the loop at the instant shown is ϵ . If you replace the loop with a coil of the same radius, but with N turns and repeat the experiment, then the emf induced around the coil at the same instant is

- (a) the same, ϵ
- (b) ϵ/N
- (c) $N\epsilon$
- (d) None of the above because it is zero.

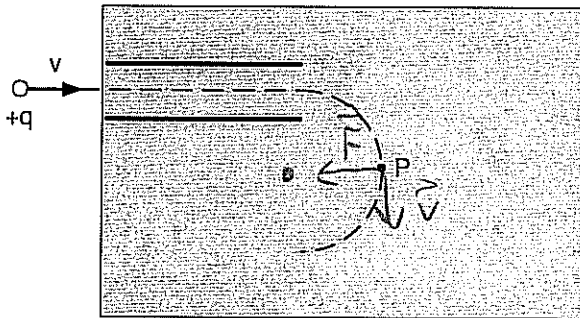
think of each loop as being in series

13. If you remove the wire loop but leave point P at its exact location and repeat the experiment with the magnet moving toward the loop, then the curly electric field at location P at this instant is

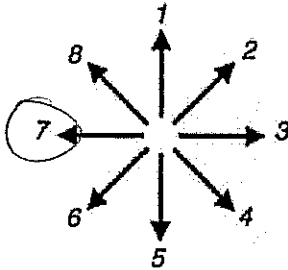
- (a) zero because the loop is required in order for there to be a curly electric field at location P.
- (b) the same as when the loop was there.
- (c) non-zero, but less than before.
- (d) non-zero, but greater than before.

current reverses the loop; \vec{E} exists regardless

Questions 14–20: A positively charged ion with charge $+e$ travels at constant velocity through charged plates. After emerging from the plates, it travels along a circular path at constant speed until it strikes a detector (which is not shown). All of this takes place in a region of uniform magnetic field, which is the shaded region shown in the picture below.



14. At point P, what is the direction of the magnetic force on the ion?



9 zero magnitude

- (a) 5
- (b) 9
- (c) 1
- (d) 6
- (e) 7

\vec{F} is toward center of circle for uniform circular motion

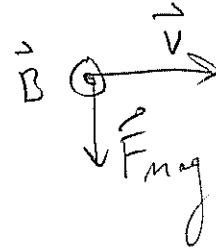
15. What is the direction of the magnetic field in the shaded region? Define $+x$ to the right, $+y$ upward toward the top of the page, and $+z$ outward toward you, perpendicular to the page.

- (a) $+x$
- (b) $-x$
- (c) $+y$
- (d) $-y$
- (e) $+z$
- (f) $-z$
- (g) None of the above because it is zero.

thumb - \vec{v}
fingers - \vec{B}
palm - $+z$ dir.

16. What is the direction of the magnetic force on the ion as it is traveling between the plates?

- (a) $+x$
- (b) $-x$
- (c) $+y$
- (d) $-y$
- (e) $+z$
- (f) $-z$
- (g) None of the above because it is zero.



17. Which plate must be negatively charged?

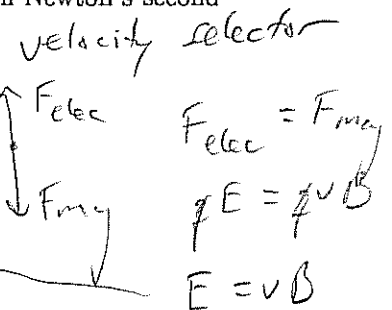
- (a) the top plate
- (b) the bottom plate
- (c) The plates must be neutral since the electric force must be zero for the ion to travel at constant velocity.



\vec{F}_{elec} on $+ion$ is toward $-plate$

18. The magnetic field is 0.5 T, the plates are separated 1 cm, and the ion's speed is 1×10^5 m/s. What is the potential difference across the plates? (Hint: start with Newton's second law.)

- (a) 5×10^4 V
- (b) 500 V
- (c) 5×10^5 V
- (d) 2000 V
- (e) 2 V



$\Delta V = Ed$
 $\Delta V = vBd = 500V$

$E = vB$

19. If the radius of the circle is 10 cm, what is the mass of the ion, in kg?

- (a) 1.67×10^{-27} kg
- (b) 2.3×10^{-27} kg
- (c) 6.7×10^{-27} kg
- (d) 3.3×10^{-25} kg
- (e) 8.0×10^{-26} kg

$$qvB = \frac{mv^2}{R}$$

$$qB = \frac{mv}{R}$$

$$m = \frac{qBR}{v} = 8 \times 10^{-26} \text{ kg}$$

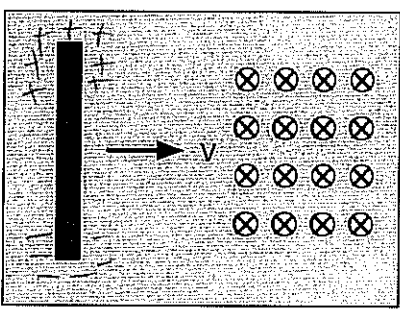
20. Of the following choices, what type of atom must this be? (This is the goal of mass spectrometry: to identify atoms by mass.)

- (a) N (14.0 g/mol)
- (b) Si (28.1 g/mol)
- (c) Ti (47 g/mol)
- (d) As (74.9 g/mol)
- (e) Au (197 g/mol)

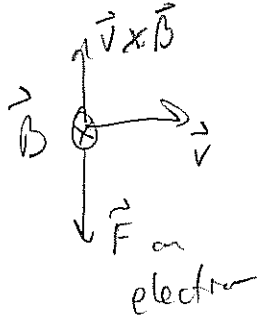
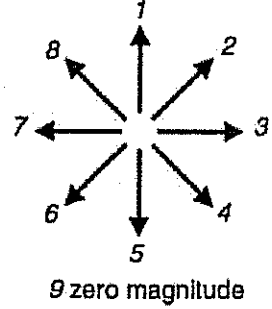
$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

$$\frac{m}{m_p} = \frac{8 \times 10^{-26} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} = 47.9$$

Questions 21–26: A wire moves at constant velocity through a region of uniform magnetic field, which is the shaded region shown below.



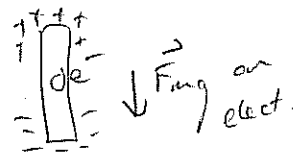
21. What is the direction of the magnetic force on an electron in the wire?



- (a) 1
- (b) 3
- (c) 7
- (d) 9
- (e) 5

22. Which end of the wire will be negatively charged?

- (a) the bottom end
- (b) the top end



23. Once the wire becomes polarized and reaches equilibrium, then the net force on a mobile electron is zero. If the magnetic field is 0.1 T and the wire is 10 cm long, what is the emf across the wire if the wire is moving at a speed of 0.5 m/s?

- (a) 0.01 V
- (b) 0.5 V
- (c) 0.02 V
- (d) 0.005 V
- (e) zero, because there will be no emf across the wire.

$$F_{elec} = F_{mag}$$

$$qE = qvB$$

$$E = vB$$

$$EL = vBL$$

$$\boxed{E = vBL} = 0.005 \text{ V}$$

24. After becoming polarized, does current flow through the wire?

- (a) yes
- (b) no

the charge is in static equilibrium so $I = 0$

25. What is the magnetic force on the wire?

- (a) zero
- (b) 0.01 N
- (c) 0.02 N
- (d) 0.05 N

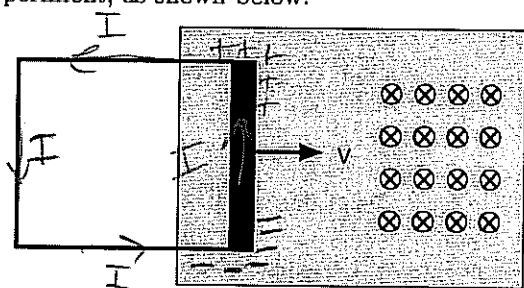
$$F_{mag} = ILB = 0$$

26. What force must be applied (perhaps by you if you pull on the wire) in order to keep the wire moving at a constant velocity?

- (a) 0.01 N
- (b) 0.02 N
- (c) 0.05 N
- (d) none, because the net force on the wire is zero and the magnetic force on the wire is zero.

If I apply a force, the wire will accelerate.

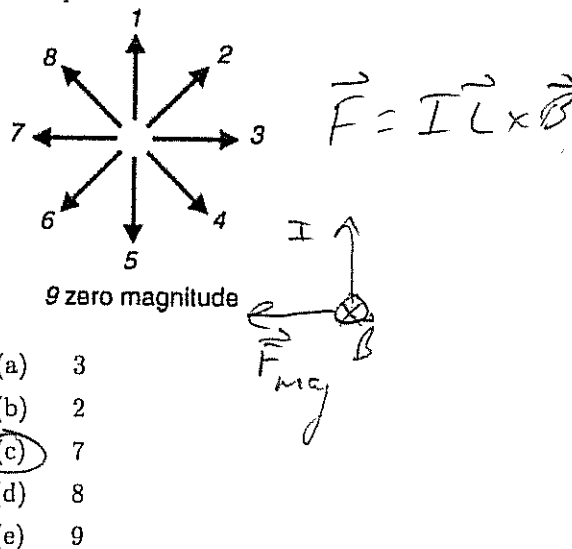
27. Now, suppose you connect the wire to other wires to form a loop and you repeat the experiment, as shown below.



In what direction does current flow around the loop?

- (a) clockwise
 (b) counterclockwise

28. In what direction is the net magnetic force on the loop?



29. If the resistance of the loop is 2Ω and if the emf is 0.001 V , what is the current through the loop?

- (a) $2 \times 10^{-3} \text{ A}$
 (b) $1 \times 10^{-3} \text{ A}$
 (c) $5 \times 10^{-4} \text{ A}$
 (d) $1 \times 10^{-4} \text{ A}$
 (e) $2 \times 10^{-4} \text{ A}$

*Small current!
 can't light a bulb.*

$$\Delta V = IR$$

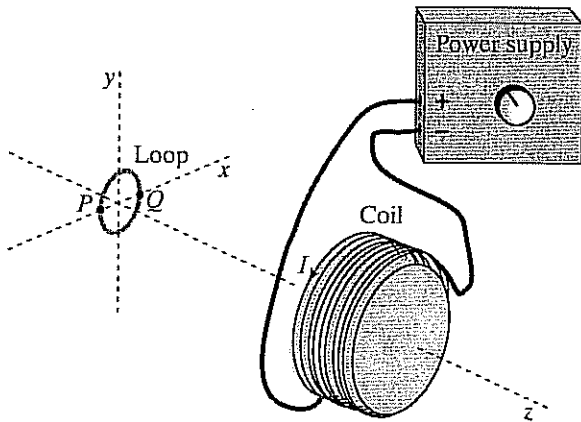
$$\mathcal{E} = IR$$

$$I = \frac{\mathcal{E}}{R} = \frac{0.001 \text{ V}}{2 \Omega}$$

=

Section 2. Problem Solving

30. A coil of wire is connected to a power supply, and a current runs in the coil. A single loop of wire is located near the coil, with its axis on the same line as the axis of the coil. The radius of the loop is 3 cm.



At time t_1 the magnetic field at the center of the loop, due to the coil, is 0.6 T and constant due to current in the coil.

- (a) At this time, what is the magnetic flux in the loop?

$$\Phi = BA = (0.6 \text{ T}) \pi R^2 = 0.0017 \text{ T}\cdot\text{m}^2$$

- (b) At this time, what is the emf induced around the loop?

$$\mathcal{E} = -\frac{d\Phi}{dt} = 0 \quad \text{since } \Phi \text{ is constant}$$

- (c) At a later time t_2 , the current in the coil begins to decrease such that the rate of change of the magnetic field at the surface of the loop is -0.25 T/s . At this instant, what is the emf around the loop? (Just report the absolute value of the emf.)

$$\frac{dB}{dt} = -0.25 \frac{\text{T}}{\text{s}}$$

$$\begin{aligned} \mathcal{E} &= -\frac{d\Phi}{dt} = -\frac{d}{dt}(BA) \\ &= -A \frac{dB}{dt} = -\pi R_{\text{loop}}^2 \left(\frac{dB}{dt}\right) \\ &= \pi (0.03 \text{ m})^2 (0.25 \frac{\text{T}}{\text{s}}) = \end{aligned}$$

- (d) What is the magnitude of the electric field at location P inside the wire?

$$\begin{aligned} \mathcal{E} &= \oint \vec{E} \cdot d\vec{l} \\ &= E \int dl = E(2\pi R) \end{aligned}$$

$$\begin{aligned} E &= \frac{\mathcal{E}}{2\pi R} \\ &= \frac{0.00707 \text{ V}}{2\pi (0.03 \text{ m})} = \boxed{0.0038 \frac{\text{V}}{\text{m}}} \end{aligned}$$

$$\boxed{7.07 \times 10^{-4} \text{ V}}$$

- (e) If the resistance of the loop is 0.5Ω , what is the current in the loop?

$$\mathcal{E} = \Delta V = IR$$

$$\mathcal{E} = IR$$

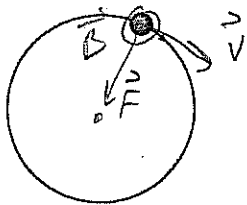
$$I = \frac{\mathcal{E}}{R} = \frac{7.07 \times 10^{-4} \text{ V}}{0.5 \Omega} = \boxed{0.0014 \text{ A}}$$

$$\boxed{0.0014 \text{ A}}$$

$$\uparrow 1.4 \text{ mA}$$

only
0.7 mV,
a small signal

31. An alpha particle enters a region of uniform magnetic field $\vec{B} = \langle 0, 0, 1.54 \rangle$ T with a velocity that is perpendicular to the field. Its speed is 5×10^5 m/s, and its path is shown below.



- (a) By applying Newton's second law for uniform circular motion, derive an expression for the radius of the alpha particle's "orbit."

$$F_{\text{net}} = \frac{mv^2}{R}$$

$$F_{\text{mg}} = F_{\text{net}}$$

$$F_{\text{mg}} = \frac{mv^2}{R}$$

$$qvB \sin 90^\circ = \frac{mv^2}{R}$$

$$qvB = \frac{mv^2}{R}$$

$$qB = \frac{mv}{R}$$

$$R = \frac{mv}{qB}$$

- (b) What is the radius of the alpha particle's path in this case?

$$R = \frac{mv}{qB}$$

$$= \frac{(6.68 \times 10^{-27} \text{ kg})(5 \times 10^5 \frac{\text{m}}{\text{s}})}{(3.2 \times 10^{-19} \text{ C})(1.54 \text{ T})}$$

$$= \boxed{0.00678 \text{ m}} = 6.78 \text{ mm}$$

$$m = 2m_p + 2m_n \approx 4m_p$$

$$= 4(1.67 \times 10^{-27} \text{ kg})$$

$$= 6.68 \times 10^{-27} \text{ kg}$$

$$q = +2(1.6 \times 10^{-19} \text{ C}) \approx 3.2 \times 10^{-19} \text{ C}$$

- (c) What is the period of the alpha particle's motion?

$$v = \frac{2\pi R}{T}$$

$$T = \frac{2\pi R}{v} = \frac{2\pi(0.00678 \text{ m})}{5 \times 10^5 \frac{\text{m}}{\text{s}}}$$

$$= \boxed{8.52 \times 10^{-8} \text{ s}}$$

$$= 85 \text{ ns} \quad \text{Wow!}$$

That's a small time interval!

Section 3. LAB

32. A speaker has three essential parts. One of them is the speaker cone, which in our case in lab was a cup. What are the other two essential parts?

a coil and a magnet (i.e. "permanent" magnet)

33. In class, we derived the force of a magnetic dipole μ_1 on another magnetic dipole μ_2 that are aligned on the same axis. The resulting expression showed that the magnetic force (of attraction or repulsion) varied as $F \propto \frac{1}{r^4}$ where r is the distance between the dipoles. By playing with the magnets, you found that a single dipole could lift up another dipole that was sitting on the table. As you moved the dipoles closer a small displacement Δx , you found that it could produce a very large increase in the force. If you decrease the distance between the two dipoles by a factor of 1/2, by what factor does the force increase?

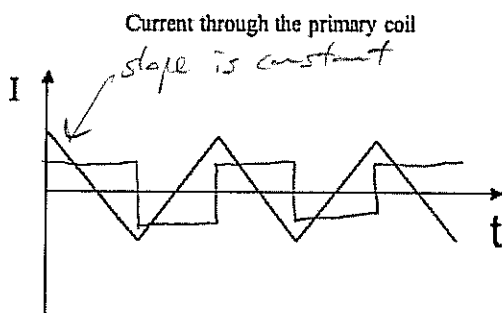
$$F \propto \frac{1}{r^4} \quad \frac{1}{2}r \text{ gives } \frac{1}{\left(\frac{1}{2}\right)^4} = \frac{1}{\frac{1}{16}} = \boxed{16F}$$

34. In what direction is current flowing in the wire on my shirt?

"down and to the left" or inward (3-D) is ok too.



35. A primary coil and a secondary coil are set up in lab with their axes aligned. The primary coil is connected to a function generator which produces an AC current in the coil as shown below.



$$\mathcal{E} = -\frac{d\Phi}{dt} = -A\frac{dB}{dt}$$

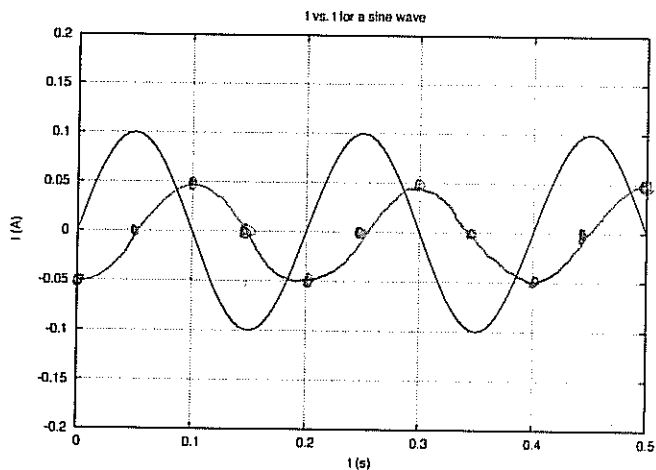
$$B \propto I, \text{ so}$$

$$\mathcal{E} \propto -\frac{dI}{dt} \quad \text{take derivative of graph}$$

On the same set of axes above, sketch the induced emf across the secondary coil. Your sketch should be neat and clear. Its shape and where it crosses the horizontal axis are important.

You can plot either the pos. derivative or neg. derivative.
Both answers are ok with me.

36. Suppose that the current in the primary coil is changed to a sinusoidal function as shown below.



$$\mathcal{E} \propto -\frac{dI}{dt}$$

plot derivative
(or neg. derivative)

On the same set of axes above, sketch the induced emf across the secondary coil. Your sketch should be neat and clear. Its shape and where it crosses the horizontal axis are important.