Physics 222, Spring 2010

Quiz 4, Form: $\boxed{A}$

Name: Key

Date: ______

Instructions

You must sketch correct pictures and vectors, you must show all calculations, and you must explain all answers for full credit. Neatness and organization is required. Points will be taken off for sloppy work.

Fundamental Concepts

Equations you should know:

1. Coulomb's law
2. Electric field of a point charge. Magnetic field of a moving point charge.
3. Relationship between electric field and electric force

Other fundamental concepts: Conservation of Charge; the Superposition Principle

Derived Results

$$\text{dipole moment} \quad p = qs$$

$$\text{Force on a neutral atom } |\vec{F}| = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{2\alpha q^2}{r^5}$$

$$\text{induced dipole moment of a neutral atom } \vec{p} = \alpha\vec{E}$$

$$|\vec{E}|_{\text{dipole}} = \frac{1}{4\pi\epsilon_0}\frac{|q|s}{r^3} \quad \text{along perpendicular bisector if } r >> s$$

$$|\vec{E}|_{\text{dipole}} = \frac{1}{4\pi\epsilon_0}\frac{2|q|s}{r^3} \quad \text{along axis of dipole if } r >> s$$

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\left(\frac{|Q|}{r(r^2+(L/2)^2)^{1/2}}\right) \quad \text{along } \perp \text{ bisector of the rod}$$

$$|\vec{E}| \approx \frac{1}{4\pi\epsilon_0}\left(\frac{2|Q|/L}{r}\right) \quad \text{along } \perp \text{ bisector of the rod if } L >> r$$

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{|Q|z}{(R^2+z^2)^{3/2}} \quad \text{along the axis of a ring}$$

$$|\vec{E}| = \frac{|Q|/A}{2\epsilon_0}\left(1 - \frac{z}{(R^2+z^2)^{1/2}}\right) \quad \text{along the axis of a disk, } A = \pi R^2$$

$$|\vec{E}| \approx \frac{|Q|/A}{2\epsilon_0}\left(1 - \frac{z}{R}\right) \quad \text{along the axis of a disk, } z << R$$

$$|\vec{E}| \approx \frac{|Q|/A}{2\epsilon_0} \quad \text{along the axis of a disk, extremely close to the disk}$$

$$|\vec{E}| \approx \frac{|Q|/A}{\epsilon_0} \quad \text{inside a capacitor, near the axis of the plates}$$

$$|\vec{E}|_{fringe} \approx \frac{|Q|/A}{2\epsilon_0}\left(\frac{s}{R}\right) \quad \text{outside a capacitor, near the axis of the plates}$$

$$|\vec{E}| = 0 \quad \text{inside a spherical shell}$$

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{|Q|}{r^2} \quad \text{outside a spherical shell}$$

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{|Q|}{R^3}r \quad \text{inside a spherical volume of uniform charge}$$

$$\Delta V = -\int_i^f \vec{E}\cdot d\vec{l} \quad \text{along any path from point i to point f} \qquad \vec{E} = -\nabla V = -< \frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z} >$$

$$\Delta V = -(E_x\Delta x + E_y\Delta y + E_z\Delta z) \quad \text{for constant electric field or small displacement}$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad \text{due to a particle or outside a charged sphere}$$

$$\vec{E}_{dielectric} = \frac{\vec{E}_{vacuum}}{K} \qquad \Delta V_{dielectric} = \frac{\Delta V_{vacuum}}{K} \qquad \epsilon = K\epsilon_0$$

$$\Delta U = q\Delta V$$

$$|\vec{B}| = \frac{\mu_0}{4\pi}\frac{LI}{r\sqrt{r^2 + (L/2)^2}} \quad \text{along } \perp \text{ bisector of a straight wire} \qquad |\vec{B}| = \frac{\mu_0}{4\pi}\frac{2I}{r} \quad r << L$$

$$|\vec{B}| = \frac{\mu_0}{4\pi}\frac{2IA}{(r^2 + R^2)^{3/2}} \quad \text{along axis of current-carrying loop} \qquad |\vec{B}| \approx \frac{\mu_0}{4\pi}\frac{2IA}{r^3} \quad \text{for loop with } r >> R$$

$$|\vec{B}| = \frac{\mu_0}{4\pi}\frac{2\mu}{r^3} \quad \text{along axis of magnetic dipole} \qquad \mu = IA \quad \text{for current-carrying loop}$$

Physical Constants

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \ \frac{\mathrm{C^2}}{\mathrm{N \cdot m^2}}$$

$$\mathrm{e} = 1.6 \times 10^{-19}\mathrm{C}$$

$$\frac{\mu_0}{4\pi} = 1 \times 10^{-7} \ \frac{\mathrm{T \cdot m}}{\mathrm{A}}$$

$$\mathrm{c} = 3 \times 10^8 \ \mathrm{m/s^2}$$

Avogradro's Number $= 6.02 \times 10^{23}$ atoms/mole

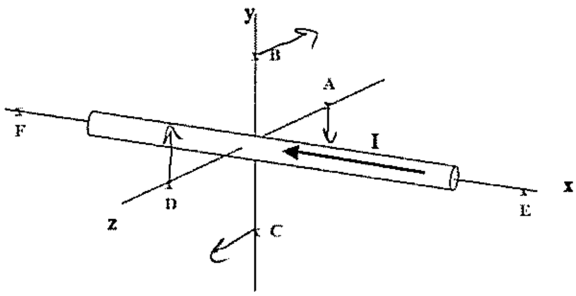
Geometry

$$C = 2\pi R \qquad A = \pi R^2$$

$$arclength = R\Delta\theta$$

Section 1. Multiple Choice

Questions 1–3: Current flows through the wire shown below. Assume the points are near the wire, compared to the length of the wire (so $r << L$).



1. What is the direction of the magnetic field at point D?

   (a) +z
   (b) −y
   (c) +x
   (d) −x
   (e) +y
   (f) −z
   (g) None of the above because the magnetic field is zero.

$\vec{B}$ curls around the wire as shown.

2. What is the direction of the magnetic field at point C?

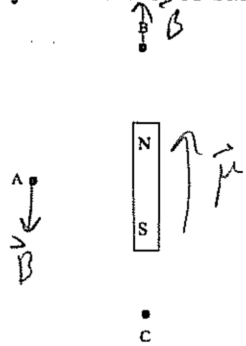
   (a) +x
   (b) −y
   (c) +y
   (d) +z
   (e) −x
   (f) −z
   (g) None of the above because the magnetic field is zero.

3. Suppose that point G is on the +z axis and is twice the distance from the wire as point D. The magnetic field at point G will be

   (a) $1/4 B_D$
   (b) $1/8 B_D$
   (c) $1/2 B_D$
   (d) $2B_D$
   (e) equal to B_D

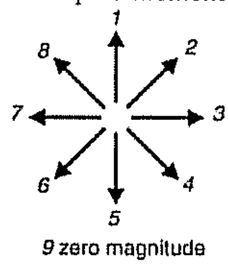
$B \propto \frac{1}{r}$

Questions 4–6: A bar magnet is shown below.



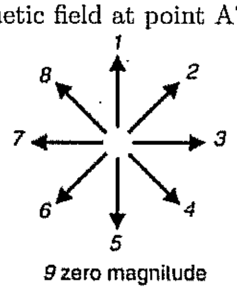
$\vec{\mu}$ is in direction of $\vec{B}$ along the axis.

4. Which arrow points in the direction of the magnetic dipole moment?



   (a) 5
   (b) 3
   (c) 1
   (d) 7
   (e) None of the above because the magnetic field is zero.

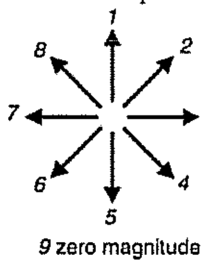
5. Which arrow points in the direction of the magnetic field at point A?



It is opposite $\vec{\mu}$

   (a) 1
   (b) 7
   (c) 5
   (d) 3
   (e) None of the above because the magnetic field is zero.

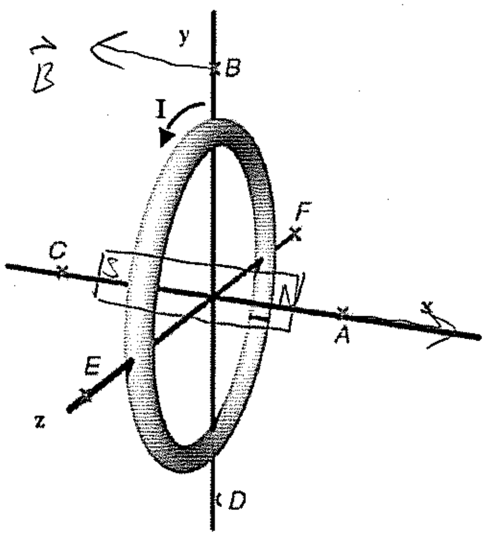
6. Which arrow points in the direction of the magnetic field at point B?



Away from N pole of magnet. In the direction of $\vec{\mu}$.

(a) 3
(b) 5
(c) 7
(d) 1
(e) None of the above because the magnetic field is zero.

Questions 7–10: Current flows in a loop of wire as shown below.



7. What is the direction of the magnetic field at point A?

(a) −y
(b) −x
(c) +y
(d) +z
(e) +x
(f) −z
(g) None of the above because the magnetic field is zero.

8. What is the direction of the magnetic field at point B?

(a) −x
(b) +y
(c) −z
(d) +x
(e) −y
(f) +z
(g) None of the above because the magnetic field is zero.

opposite magnetic field along axis of loop

9. What is the direction of the dipole moment vector for the loop?

(a) −x
(b) +y
(c) −z
(d) +x
(e) −y
(f) +z
(g) None of the above because the magnetic field is zero.

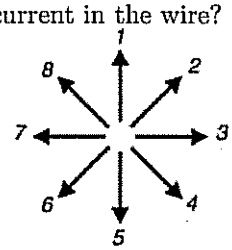
Same as $\vec{B}$ along the axis

10. The loop can be modeled as a bar magnet (i.e. magnetic dipole) because it creates a similar pattern of magnetic field. Sketch a bar magnet for the loop that is centered at the origin and creates the same magnetic field at all points around the loop. Label the bar magnet's poles with N and S. The South pole label is on what axis?

(a) −x
(b) +y
(c) −z
(d) +x
(e) −y
(f) +z
(g) None of the above because the magnetic field is zero.

Questions 11–12: A *side view* of a coil of wire with current I flowing through the coil is shown below. (Note: the battery is not shown.) As a result, the compass deflects 20 ° west of north.

11. Which arrow points in the direction of the magnetic field at the location of the compass due to current in the wire?



9 zero magnitude

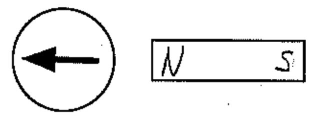
Needle deflects westward.

(a) 3
(b) 8
(c) 1
(d) 5
(e) 7

12. In what direction is current flowing at the top of the loop?

(a) into the page (–z direction)
(b) out of the page (+z direction)

13. A compass is brought near to the bar magnet as shown below. Since the magnetic field of the magnet is large compared to Earth at this location, Earth's magnetic field is negligible and the compass points in the direction of the magnetic field of the magnet. Label the magnet's N and S poles. Which side of the magnet is the N pole?



(a) the right side
(b) the left side

14. The magnet in the previous question is a neodymium bar magnet with magnetic dipole moment μ, like you used in lab. If you stick two of the magnets together, then the magnetic dipole moment of the system is

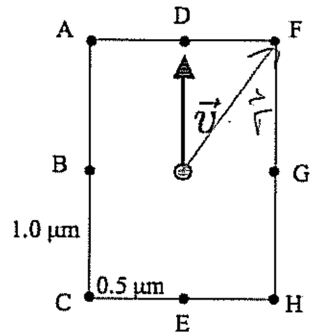
(a) $(1/2)\mu$
(b) 2μ
(c) $(1/4)\mu$
(d) the same, μ
(e) 4μ

15. Suppose that magnetic field at the location r along the axis of a tiny neodymium magnet is B. If you stick 3 of these magnets together and measure the magnetic field at the same location r, then the magnetic field will be

(a) 27B
(b) 3B
(c) the same, B
(d) (1/27)B
(e) (1/3)B

Section 2. Problem Solving

16. An *electron* moves with a speed of 1×10^5 m/s in the direction shown below.



$$\vec{r} = \langle 0.5, 1.0, 0 \rangle \times 10^{-6}\ m$$

$$|\vec{r}| = 1.118 \times 10^{-6}\ m$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \langle 0.447, 0.894, 0 \rangle$$

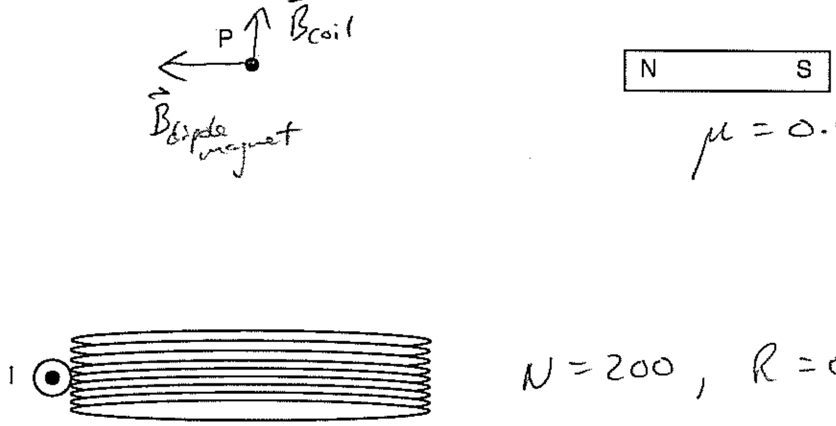
The box is merely shown to help you determine the position vector of various points. Define the origin to be the location of the electron at this instant. What is the magnetic field at point F?

$$\vec{B} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \hat{r}}{|\vec{r}|^2} \qquad \vec{v} = \langle 0, 1 \times 10^5 \tfrac{m}{s}, 0 \rangle$$

$$\vec{B} = \left(1 \times 10^{-7} \frac{T m^2}{C \frac{m}{s}}\right)\left(-1.6 \times 10^{-19} C\right)\left(\frac{\langle 0, 1 \times 10^5, 0 \rangle \frac{m}{s} \times \langle 0.447, 0.894, 0 \rangle}{(1.118 \times 10^{-6} m)^2}\right)$$

$$= \langle 0, 0, 5.72 \times 10^{-10} \rangle T$$

17. A coil and bar magnet are oriented as shown below. Point P is 20 cm from the center of the coil and 20 cm from the center of the bar magnet. The bar magnet has a magnetic dipole moment 0.4 A m^2. The coil is NOT drawn to scale in the picture. It is actually quite thin and is made of 200 turns of very thin wire. Its radius is only 0.01 m, which can be considered small compared to the distance to point P. The current in the coil is 6.4 A and is directed outward at the left side of the coil and inward at the right side of the coil.



$\mu = 0.4\ A \cdot m^2$

$N = 200, \quad R = 0.01m, \quad I = 6.4A$

What is the net magnetic field at point P? (Remember that famous superhero "Superposition.")

$\vec{B}_{coil}$ is in the $+y$ direction with magnitude:

$$|\vec{B}| \approx \frac{\mu_0}{4\pi} \frac{2NIA}{r^3} = \frac{\mu_0}{4\pi} \frac{2(200)(6.4A)\pi(0.01m)^2}{(0.2m)^3}$$

$$= 1.01 \times 10^{-5}\ T$$

$$\vec{B}_{coil} = \langle 0, 1.01\times 10^{-5}, 0\rangle\ T$$

$\vec{B}_{magnet}$ is in the $-x$ direction with magnitude:

$$|\vec{B}| = \frac{\mu_0}{4\pi} \frac{2\mu}{r^3}$$

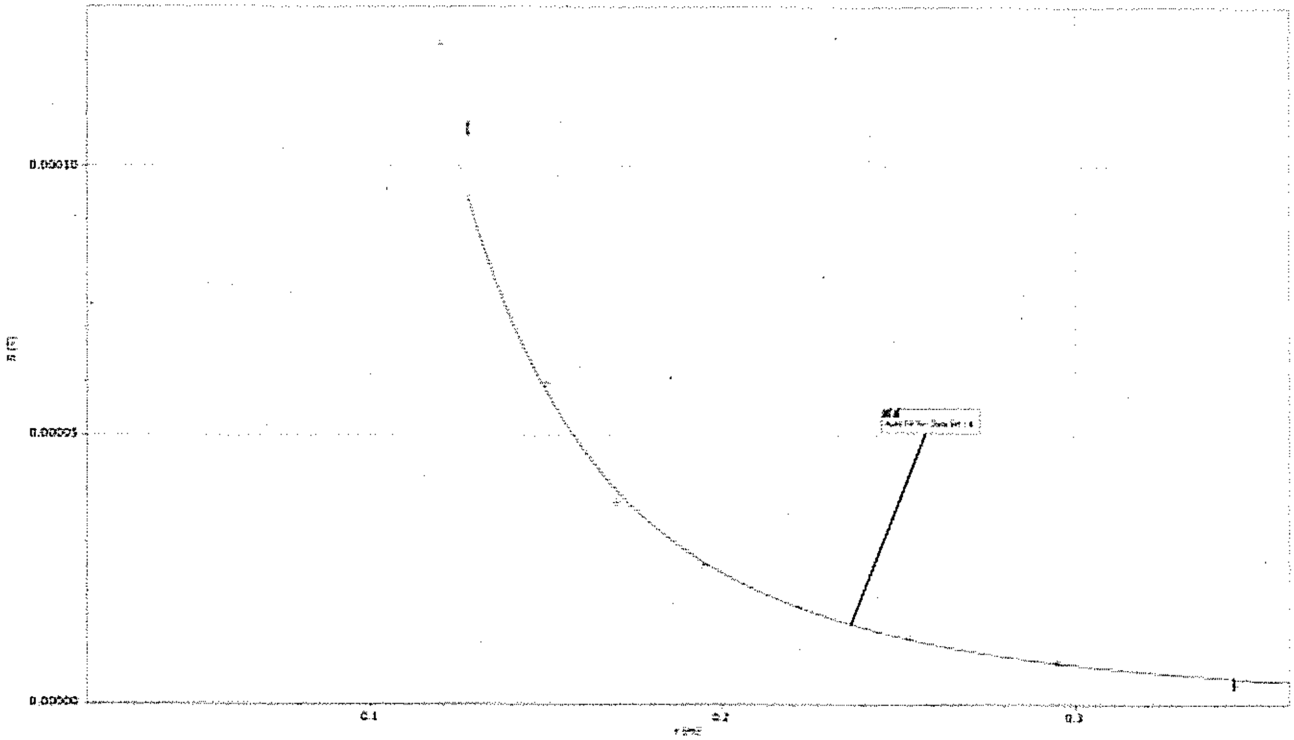
$$= \frac{\mu_0}{4\pi} \frac{2(0.4\ A\cdot m^2)}{(0.2m)^3} = 1 \times 10^{-5}\ T$$

$$\vec{B}_{magnet} = \langle -1\times 10^{-5}, 0, 0\rangle\ T$$

$$\vec{B}_{net} = \vec{B}_{magnet} + \vec{B}_{coil} = \langle -1\times 10^{-5}, 1\times 10^{-5}, 0\rangle\ T$$

Section 3. LAB

18. (a) In the lab, you measure the magnetic field as a function of distance along the axis of a bar magnet. You graph B vs. r and obtain the graph shown below.



If you fit a function of the form $B = \frac{C}{r^3}$ and the constant C is 2×10^{-7} T m^3, what is the dipole moment of the magnet?

$|\vec{B}| \approx \frac{\mu_0}{4\pi}\frac{2\mu}{r^3}$ thus $\frac{\mu_0}{4\pi} 2\mu = C$

$\mu = \frac{(2 \times 10^{-7})}{(1 \times 10^{-7})2} = 1 \ A \cdot m^2$

(b) Suppose that you repeat the experiment with a coil of 20 turns and radius 0.015 m. From your data, you determine that dipole moment of the coil is ~~5.6×10^{-9} T m^3~~. What is the current in the coil?

$0.028 \ A \cdot m^2$

$\mu = NIA$

$I = \frac{\mu}{NA} = \frac{0.028}{(20)\pi(0.015m)^2} = 1.98 \ A$

(c) In the lab, you found that four neodymium bar magnets were much stronger than your current-carrying coil. When we talk about how "strong" a magnet is, what physical property of the magnet are we referring to? (In other words, what property of a magnet or coil determines how "strong" it is.)

The dipole moment μ characterizes the "strength" of the magnet.

(d) In the lab, you wound a 1-m long wire into a coil and you connected its ends to a battery. Name 3 distinct changes you could make in this experiment to increase the magnetic dipole moment of the coil and thus create a larger magnetic field at a given distance from the coil.

$B_{coil} \approx \frac{\mu_0}{4\pi} \frac{2NIA}{r^3}$ along the axis, where $\mu = NIA$

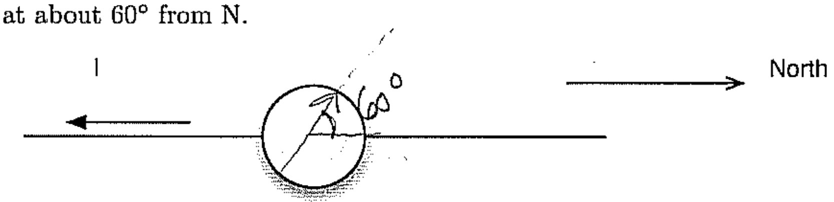
To increase μ, you can:

(1) Increase the number of turns, N (more loops!)

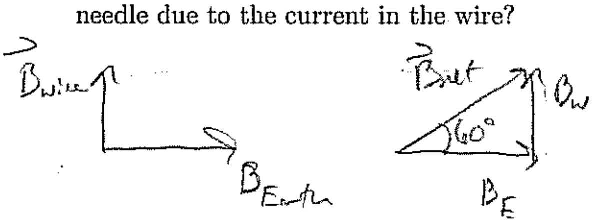
(2) Increase the current, I (more batteries to give larger current!!)

(3) larger radius (larger area, A)

(e) A compass is placed on top of a wire as shown below. Sketch the compass needle if it is deflected at about 60° from N.



(f) For the question above, what is the magnitude of the magnetic field at the location of the compass needle due to the current in the wire?



$\tan 60° = \frac{B_W}{B_E}$ $\quad B_E \approx 2\times10^{-5}\,T$

$B_w = B_E \tan 60 = 3.46\times10^{-5}\,T$

(g) For the question above, what is the current in the wire if the compass needle is 2 mm from the wire?

$r = 0.002\,m$

$$|\vec{B}|_{long,\,straight\,wire} = \frac{\mu_0}{4\pi}\frac{2I}{r} = 3.46\times10^{-5}\,T$$

$$I = \frac{(3.46\times10^{-5}\,T)(0.002\,m)}{(1\times10^{-7})(2)} = \boxed{0.346\,A}$$