Physics 222, Spring ~~2009~~ 2010
Quiz 3, Form: A

Name: Key

Date:

Instructions

You must sketch correct pictures and vectors, you must show all calculations, and you must explain all answers for full credit. Neatness and organization is required. Points will be taken off for sloppy work.

Fundamental Concepts

Equations you should know:

1. Coulomb's law
2. Electric field of a point charge
3. Relationship between electric field and electric force

Other fundamental concepts: Conservation of Charge; the Superposition Principle

Derived Results

dipole moment $p = qs$

Force on a neutral atom $|\vec{F}| = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{2\alpha q^2}{r^5}$

induced dipole moment of a neutral atom $\vec{p} = \alpha\vec{E}$

$|\vec{E}|_{\text{dipole}} = \frac{1}{4\pi\epsilon_0}\frac{|q|s}{r^3}$ along perpendicular bisector if $r >> s$

$|\vec{E}|_{\text{dipole}} = \frac{1}{4\pi\epsilon_0}\frac{2|q|s}{r^3}$ along axis of dipole if $r >> s$

$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\left(\frac{|Q|}{r(r^2 + (L/2)^2)^{1/2}}\right)$ along $\perp$ bisector of the rod

$|\vec{E}| \approx \frac{1}{4\pi\epsilon_0}\left(\frac{2|Q|/L}{r}\right)$ along $\perp$ bisector of the rod if $L >> r$

$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{|Q|z}{(R^2 + z^2)^{3/2}}$ along the axis of a ring

$|\vec{E}| = \frac{|Q|/A}{2\epsilon_0}\left(1 - \frac{z}{(R^2 + z^2)^{1/2}}\right)$ along the axis of a disk, $A = \pi R^2$

$|\vec{E}| \approx \frac{|Q|/A}{2\epsilon_0}\left(1 - \frac{z}{R}\right)$ along the axis of a disk, $z << R$

$|\vec{E}| \approx \frac{|Q|/A}{2\epsilon_0}$ along the axis of a disk, extremely close to the disk

$$|\vec{E}| \approx \frac{|Q|/A}{\epsilon_0}$$ inside a capacitor, near the axis of the plates

$$|\vec{E}|_{fringe} \approx \frac{|Q|/A}{2\epsilon_0}\left(\frac{s}{R}\right)$$ outside a capacitor, near the axis of the plates

$$|\vec{E}| = 0$$ inside a spherical shell

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{|Q|}{r^2}$$ outside a spherical shell

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{|Q|}{R^3}r$$ inside a spherical volume of uniform charge

$$\Delta V = -\int_i^f \vec{E}\cdot d\vec{l}$$ along any path from point i to point f $$\vec{E} = -\nabla V = -< \frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z} >$$

$$\Delta V = -(E_x\Delta x + E_y\Delta y + E_z\Delta z)$$ for constant electric field or small displacement

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$ due to a particle or outside a charged sphere

$$\vec{E}_{dielectric} = \frac{\vec{E}_{vacuum}}{K} \qquad \Delta V_{dielectric} = \frac{\Delta V_{vacuum}}{K} \qquad \epsilon = K\epsilon_0$$

$$\Delta U = q\Delta V$$

Physical Constants

$$\frac{1}{4\pi\epsilon_0} = 9\times 10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}$$

$$\epsilon_0 = 8.85\times 10^{-12}\ \frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}$$

$$e = 1.6\times 10^{-19}\mathrm{C}$$

$$c = 3\times 10^8\ \mathrm{m/s^2}$$

Avogradro's Number $= 6.02\times 10^{23}$ atoms/mole

Geometry

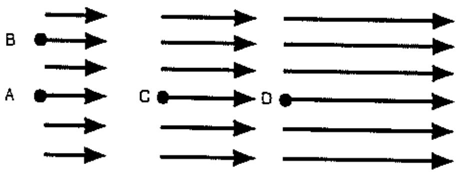
$$C = 2\pi R$$

$$arclength = R\Delta\theta$$

$$A = \pi R^2$$

Section 1. Multiple Choice

Questions 1–3: The image below shows a region in space where the electric field is NOT uniform but instead varies with location x.



Suppose that the electric field varies quadratically with x such that

$$E_x = (1 \times 10^4 \text{ V/m}^3)x^2 + 200 \text{ V/m}$$

In other words, $E_x = (1 \times 10^4)x^2 + 200$ where E is in V/m and x is in meters.

1. Which point is at a lowest electric potential V?

   (a) A
   (b) B
   (c) C
   (d) D
   (e) Both A and B are at the lowest potential.

2. If point C and point D are 5 cm apart, what is $V_C - V_D$? (Note: attempting to answer without using integration can be damaging to one's grade. Consider point C to be at $x = 0$ and point D to be at $x = 0.05$ m when doing the integration.)

   (a) 25 V
   (b) 225 V
   (c) 0.42 V
   (d) 1000 V
   (e) 10.4 V

3. If point A and point B are 5 cm apart, what is $V_B - V_A$?

   (a) 25 V
   (b) 225 V
   (c) 0 V
   (d) 1000 V
   (e) 10.4 V

4. Suppose that an electron moves from point i at $V_i = 0$ V to point f at $V_f = 120$ V. During this motion, the electron's change in potential energy ΔU is

   (a) positive
   (b) negative
   (c) zero

5. For the electron in the previous question, the electron's change in kinetic energy ΔK is

   (a) positive
   (b) negative
   (c) zero

6. Suppose that an electron moves from point i at $V_i = 2V$ to point f at $V_f = 10V$. What is its change in kinetic energy, in joules?

   (a) 8 J
   (b) 1.28×10^{-18} J
   (c) -1.28×10^{-18} J
   (d) -8 J
   (e) 3.2×10^{-19} J

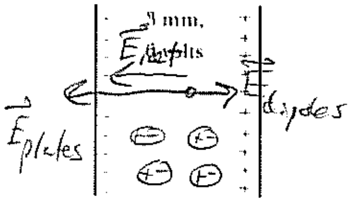
7. For the electron in the previous question, what is its change in kinetic energy in electron-volts?

   (a) 10 eV
   (b) 2 eV
   (c) -2 eV
   (d) -8 eV
   (e) 8 eV

8. For a conductor in static equilibrium, the electric potential at all points in the conductor is

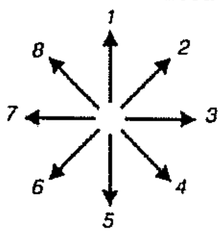
   (a) constant.
   (b) always zero.

Questions 9–13: Isolated, charged capacitor plates are shown below. The potential difference across the plates, when there is a vacuum between the plates, is 6 V.



Suppose that you insert an insulator of dielectric constant $K = 10$ that fills the gap between the plates.

9. What is the direction of the electric field due to the induced dipoles in the insulator?



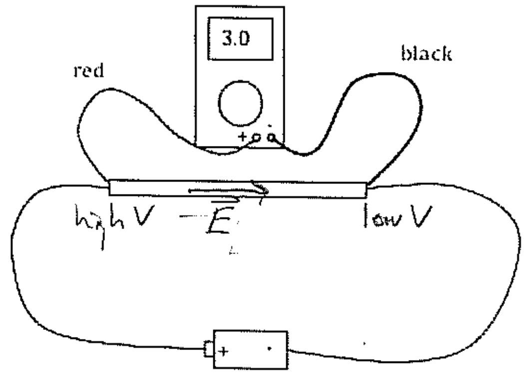
9 zero magnitude

   (a) 1
   (b) 7
   (c) 5
   (d) 3
   (e) 9

10. The net electric field after the insulator is inserted is
   (a) greater than the electric field due to the plates.
   (b) less than the electric field due to the plates.
   (c) equal to the electric field due to the plates.
   (d) zero.

11. What is the potential difference across the plates after inserting the insulator?
   (a) 60 V
   (b) 16 V
   (c) 6 V
   (d) 0.6 V
   (e) 4 V

12. Suppose that in a given circuit, the electric field on the inside of a 0.1-m long wire (filament) is uniform. (Note: this means that the electrons in the wire are NOT in equilibrium.) If the magnitude of the potential difference across the length of the wire is measured with a voltmeter to be 3.0 V, what is the magnitude of the electric field within the wire?

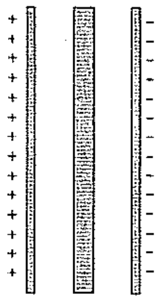


   (a) 300 V/m
   (b) 3.0 V/m
   (c) 30 V/m
   (d) 0.30 V/m
   (e) zero

13. For the wire in the previous question, the electric field within the wire is
   (a) to the right.
   (b) to the left.
   (c) is zero, so it has no direction.

14. In what direction is the force on a mobile electron by the electric field within the wire?
   (a) to the right
   (b) to the left
   (c) It is zero, so it has no direction.

Questions 15–18: A capacitor with a vacuum between its plates is initially charged so that the potential difference across the plates is 3 V. The plate separation is 3 mm and is small compared to the dimensions of the plates. Then a conductor of width 1 mm is inserted between the plates as shown below.



15. What is the magnitude of the net electric field within the gaps between the plates and the conductor?

(a) 90 V/m
(b) 3000 V/m
(c) 2000 V/m
(d) 1000 V/m
(e) 0

16. What is the magnitude of the net electric field within the conductor?

(a) 90 V/m
(b) 3000 V/m
(c) 2000 V/m
(d) 1000 V/m
(e) 0

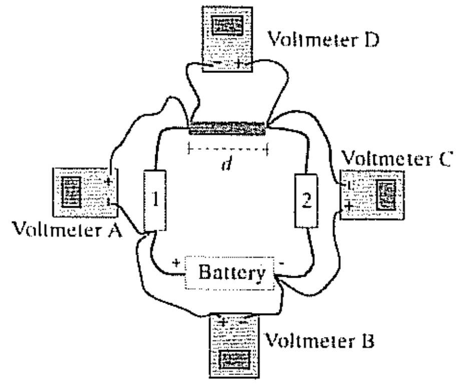
17. What is the magnitude of the potential difference across the conductor?

(a) zero
(b) 1 V
(c) 2 V
(d) 3 V
(e) 9 V

18. What is the potential difference across the plates?

(a) zero
(b) 1 V
(c) 2 V
(d) 3 V
(e) 9 V

19. A voltmeter is connected to a circuit as shown below.



The voltmeter connected to the battery reads a potential difference of 9.0 V. Voltmeter A reads −2.0 V and voltmeter D reads −4.0 V. What does voltmeter C read?

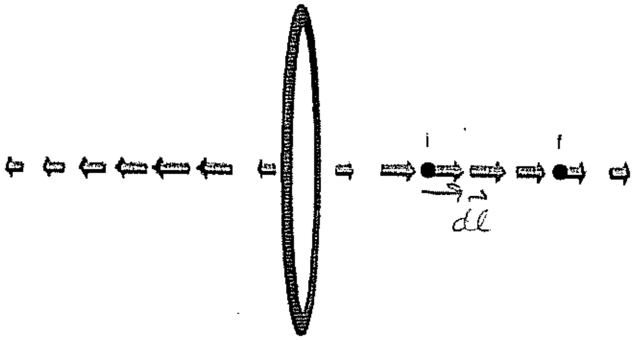
(a) 15.0 V
(b) -6.0 V
(c) -15.0 V
(d) -3.0 V
(e) 6.0

20. For the previous question, if an electron moves through the battery from the + terminal to the − terminal (the electron would move toward the right in the picture, as it moves through the battery), does the electron gain potential energy or lose potential energy? (Note: a particle doesn't have potential energy, so it's the system of the electron and electric field that has potential energy.)

(a) It gains potential energy.
(b) It loses potential energy.
(c) Neither because its potential energy remains constant.

Section 2. Problem Solving

21. The electric field along the axis of a uniformly charged ring is shown below.



If r is the distance from the center of the ring to a point along the axis of the ring, then the electric field along the axis of the ring is

$$E_r = \frac{1}{4\pi\epsilon_0}\frac{|Q|r}{(R^2+r^2)^{3/2}}$$

where Q is the charge of the ring and R is the radius of the ring. What is the potential difference $V_f - V_i$ along a path from point i to point f?

$$\Delta V = -\int_i^f \vec{E}\cdot d\vec{l}$$

$$= -\int_{r_i}^{r_f} E_r\,dr = -\int_{r_i}^{r_f} \frac{1}{4\pi\epsilon_0}\frac{|Q|r}{(R^2+r^2)^{3/2}}\,dr$$

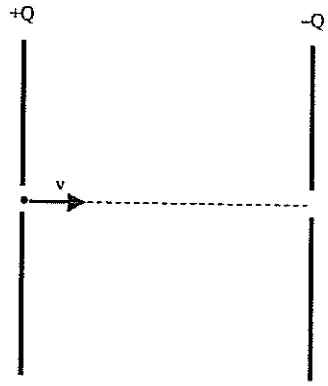
$$= -\frac{1}{4\pi\epsilon_0}|Q|\left[\frac{-1}{(R^2+r^2)^{1/2}}\right]_{r_i}^{r_f}$$

$$\boxed{\Delta V = \frac{|Q|}{4\pi\epsilon_0}\left(\frac{1}{(R^2+r_f^2)^{1/2}} - \frac{1}{(R^2+r_i^2)^{1/2}}\right)}$$

Note that for a positively charged ring, ΔV is negative, just as expected. In general,

$$\boxed{V_{\text{ring on axis}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{(R^2+r^2)^{1/2}}}$$

22. A positively charged oxygen ion (with net charge $+1e$) is accelerated by two closely spaced, oppositely charged plates, as shown below.



M_{oxygen} ? $\left(\frac{16\ g}{mol}\right)\left(\frac{1\ mol}{6.0 \times 10^{23}\ atoms}\right)$

$= \left(2.67 \times 10^{-23}\ g\right)\left(\frac{1\ kg}{1000\ g}\right)$

$= 2.67 \times 10^{-26}\ kg$

The atom has a speed of 1×10^4 m/s when it enters a slit in the positively charged plate. After traveling for 2 mm, it passes through a slit in the negatively charged plate. If the goal is to make the atom reach a speed of 1×10^6 m/s, what must be the potential difference across the plates? (Note: the plate separation is small compared to the dimensions of the plates. Also, the molar mass of oxygen is 16 g/mol.)

The system is the atom and plates. It is a closed system, so $\Delta E = 0$.

$$\Delta U + \Delta K = 0$$

$$q \Delta V + \left(\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2\right) = 0$$

Solve for ΔV.

$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2} m (1 \times 10^4)^2 = 1.33 \times 10^{-18}\ J$ ← negligible actually.

$K_f = \frac{1}{2} m v_f^2 = \frac{1}{2} m (1 \times 10^6)^2 = 1.33 \times 10^{-14}\ J$

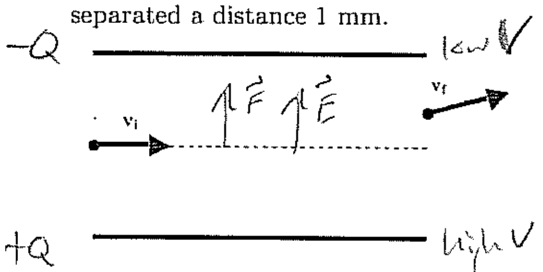
$$q \Delta V = -\Delta K$$

$$\Delta V = -\frac{\Delta K}{q} = -\frac{\Delta K}{(-e)} = \frac{\Delta K}{e} = \frac{1.33 \times 10^{-14}\ J - 1.33 \times 10^{-18}\ J}{1.6 \times 10^{-19}\ C}$$

$= 8.31 \times 10^4\ V$ Wow! that's 83,100 volts!

Accelerating particles requires high voltage!

23. A proton enters a region of uniform electric field between two closely spaced, oppositely charged plates as shown below with an initial speed of 1.0×10^6 m/s. Upon exiting the region, it has been deflected upward. The horizontal displacement of the proton through the plates is 5 cm, and the plates are separated a distance 1 mm.



$m = 1.67 \times 10^{-27}$ kg

If the voltage across the plates is 10 V, what is the vertical displacement of the proton as it travels through the region between the plates? (Note: if the displacement is great enough, it can actually collide with the plate before it gets to the end of the plates. If this occurs, then say so.)

$$F_{net,y} = \frac{m \Delta V_y}{\Delta t}$$

$$q E_y = m \frac{V_{fy} - V_{iy}}{\Delta t} \quad (V_{iy} = 0)$$

$$V_x = \frac{\Delta x}{\Delta t} = \text{constant!}$$

$$\Delta t = \frac{\Delta x}{V_x} = \frac{0.05\,m}{1 \times 10^6 \frac{m}{s}} = 5 \times 10^{-8} s$$

$$V_{fy} = \frac{q E_y \Delta t}{m} = \frac{q \frac{|\Delta V|}{s} \Delta t}{m} = \frac{(1.6 \times 10^{-19} C)\left(\frac{10 V}{0.001 m}\right)(5 \times 10^{-8} s)}{1.67 \times 10^{-27} kg}$$

$$= 4.79 \times 10^4 \frac{m}{s}$$

$$V_{avg,y} = \frac{V_{iy} + V_{fy}}{2} = \frac{4.79 \times 10^4 \frac{m}{s}}{2} = 2.395 \times 10^4 \frac{m}{s}$$

$$V_{avg,y} = \frac{\Delta y}{\Delta t}$$

$$\Delta y = V_{avg,y} \Delta t = (2.395 \times 10^4 \frac{m}{s})(5 \times 10^{-8} s)$$

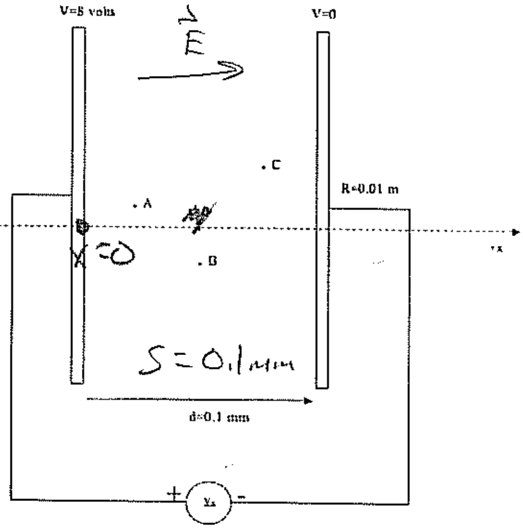
$$\boxed{\Delta y = 1.20 \times 10^{-3} m = 1.2 \text{ mm}}$$

The plates are only separated 1mm. The max displacement possible is $\frac{1}{2}$(1mm) = 0.5mm. Thus, the atom hits the top plate before reaching the other side. The experimenter needs to turn down the voltage across the plates. It's too big!

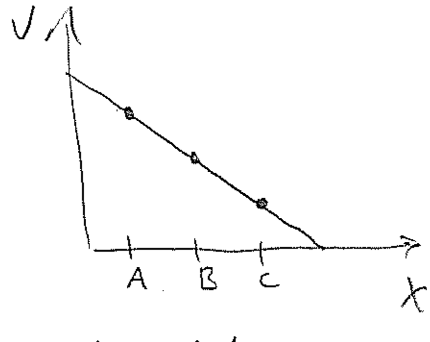
Section 3. LAB

A capacitor is connected to an 8-volt battery that maintains the plates at 8 V and 0 V, respectively, as shown below.

Put $x=0$ at left plate. It's easier to think about.



V is linear since $\vec{E}$ is constant



24. (a) What is the potential V_A at point A? from the right plate, $x_A = \frac{3}{4}s$

so $V_A = \frac{3}{4} V_{total} = \frac{3}{4}(8\text{ V}) = \boxed{6\text{ V}}$

(b) What is the potential V_B at point B?

$V_B = \frac{1}{2}(V_{total}) = \frac{1}{2}(8) = 4\text{ V}$

(c) What is the potential V_C at point C?

$V_C = \frac{1}{4}(V_{total}) = \frac{1}{4}(8) = 2\text{ V}$

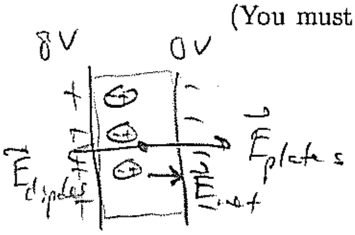
(d) What is the electric field at all points between the plates?

$|\Delta V| = Es \qquad E = \frac{\Delta V}{s} = \frac{8\text{ V}}{0.1\times10^{-3}\text{ m}} = 8\times10^{4}\ \frac{\text{V}}{\text{m}}$

(e) If you keep the battery connected to the capacitor and you fill the space between the plates with an insulator, will the magnitude of the net electric field between the plates increase, decrease, or remain the same? (You must explain your reasoning. A correct answer with incorrect reasoning will not receive credit.)

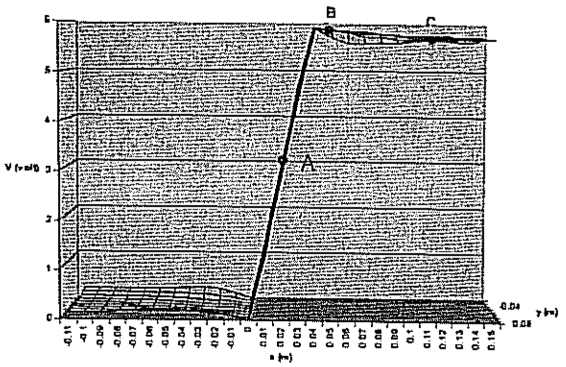
$\Delta V = \int -\vec{E}\cdot d\vec{l} = -E_x s$. Note that ΔV is the same and s is the same. So, E_x is the same! $\vec{E}_{net}$ is the same!

(f) If you keep the battery connected to the capacitor and you fill the space between the plates with an insulator, will the magnitude of the charge on each plate increase, decrease, or remain the same? (You must explain your reasoning. A correct answer with incorrect reasoning will not receive credit.)



If $\vec{E}_{net}$ stays the same, then $\vec{E}_{plates}$ must have increased. Since $E_{plates} = \frac{Q/A}{\epsilon_0}$, there must be more charge on the plates. The battery caused more charge to "pile" up on the plates.

25. A 2-D surface plot of V as a function of x and y for a capacitor is shown below. Note that it is similar to what you measured in lab.



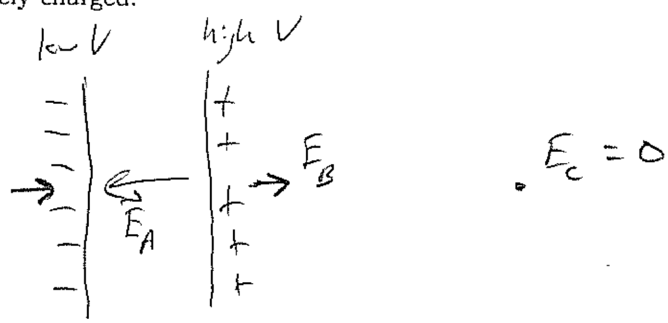
$E_x = -\frac{dV}{dx} = -\text{slope}$

A B C

$|E_x|_A > |E_x|_B > |E_x|_C$

(a) Rank the $|E_x|$ at the three points: A, B, and C on the plot.

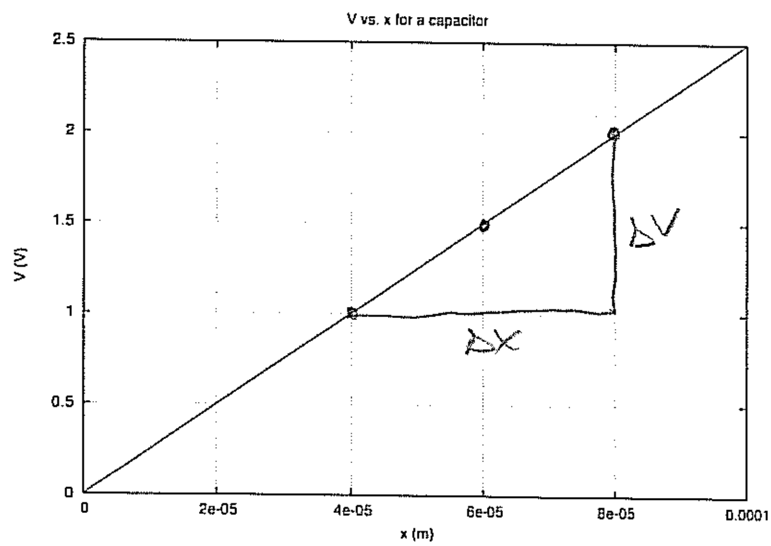
(b) Sketch a picture of the capacitor and show which capacitor plate is positively charged and which one is negatively charged.



E_B is very small compared to E_A.

(c) On your picture above, sketch vectors for the electric field inside the plates and the electric field at a point just outside each plate (but near the plate, like point B in the surface plot.)

26. A graph of V vs. x is shown below. What is the electric field at the location $x = 6 \times 10^{-5}$ m?



$E_x = -\text{slope}$

$= -\frac{\Delta V}{\Delta x}$

$= \frac{-1.0\text{ V}}{(8\times10^{-5} - 4\times10^{-5})\text{m}}$

$= 2.5 \times 10^{4}\ \frac{\text{V}}{\text{m}}$