Physics 222, Spring 2009
Quiz 2, Form: $\boxed{\text{A}}$

Name: Key
Date: ______

Instructions

You must sketch correct pictures and vectors, you must show all calculations, and you must explain all answers for full credit. Neatness and organization is required. Points will be taken off for sloppy work.

Fundamental Concepts

Equations you should know:

1. Coulomb's law
2. Electric field of a point charge
3. Relationship between electric field and electric force

Other fundamental concepts: Conservation of Charge; the Superposition Principle

Derived Results

$$\text{dipole moment} \quad p = qs$$

$$\text{Force on a neutral atom } |\vec{F}| = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{2\alpha q^2}{r^5}$$

$$\text{induced dipole moment of a neutral atom } \vec{p} = \alpha\vec{E}$$

$$|\vec{E}|_{\text{dipole}} = \frac{1}{4\pi\epsilon_0}\frac{|q|s}{r^3} \quad \text{along perpendicular bisector if } r >> s$$

$$|\vec{E}|_{\text{dipole}} = \frac{1}{4\pi\epsilon_0}\frac{2|q|s}{r^3} \quad \text{along axis of dipole if } r >> s$$

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\left(\frac{|Q|}{r(r^2+(L/2)^2)^{1/2}}\right) \quad \text{along } \perp \text{ bisector of the rod}$$

$$|\vec{E}| \approx \frac{1}{4\pi\epsilon_0}\left(\frac{2|Q|/L}{r}\right) \quad \text{along } \perp \text{ bisector of the rod if } L >> r$$

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{|Q|z}{(R^2+z^2)^{3/2}} \quad \text{along the axis of a ring}$$

$$|\vec{E}| = \frac{|Q|/A}{2\epsilon_0}\left(1 - \frac{z}{(R^2+z^2)^{1/2}}\right) \quad \text{along the axis of a disc, } A = \pi R^2$$

$$|\vec{E}| \approx \frac{|Q|/A}{2\epsilon_0}\left(1 - \frac{z}{R}\right) \quad \text{along the axis of a disc, } z << R$$

$$|\vec{E}| \approx \frac{|Q|/A}{2\epsilon_0} \quad \text{along the axis of a disc, extremely close to the disc}$$

$$|\vec{E}| \approx \frac{|Q|/A}{\epsilon_0} \quad \text{inside a capacitor, near the axis of the plates}$$

$$|\vec{E}|_{fringe} \approx \frac{|Q|/A}{2\epsilon_0}\left(\frac{s}{R}\right) \quad \text{outside a capacitor, near the axis of the plates}$$

$$|\vec{E}| = 0 \quad \text{inside a spherical shell}$$

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{|Q|}{r^2} \quad \text{outside a spherical shell}$$

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{|Q|}{R^3}r \quad \text{inside a spherical volume of uniform charge}$$

Physical Constants

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \ \frac{\mathrm{C^2}}{\mathrm{N \cdot m^2}}$$

$$\mathrm{e} = 1.6 \times 10^{-19}\mathrm{C}$$

$$\mathrm{c} = 3 \times 10^8 \ \mathrm{m/s^2}$$

Avogradro's Number $= 6.02 \times 10^{23}$ atoms/mole

Geometry

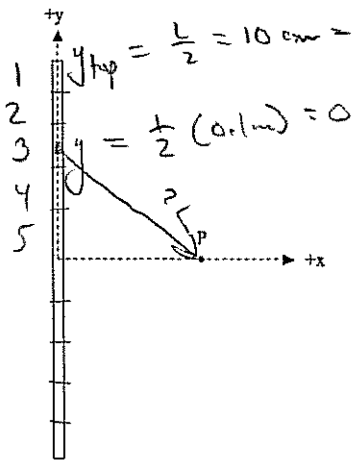
$$C = 2\pi R$$

$$arclength = R\Delta\theta$$

$$A = \pi R^2$$

Section 1. Multiple Choice

A 20-cm long thin rod has a uniform charge of 1 μC as shown below.



$y_{top} = \frac{L}{2} = 10\text{ cm} = 0.1\text{ m}$

$y = \frac{1}{2}(0.1\text{ m}) = 0.05\text{ m}$

We want to calculate the electric field at point P numerically by breaking the rod into a total of 10 pieces ($N = 10$) of equal length.

1. What is the charge of each piece?

   (a) 1 C
   (b) 1×10^{-6} C
   (c) 1×10^{-10} C
   (d) 0.1 C
   (e) 1×10^{-7} C

   $dq = \frac{Q}{N} = \frac{1\times10^{-6}\text{ C}}{10} = 1\times10^{-7}\text{ C}$

2. What is the length of each piece

   (a) 0.2 m
   (b) 20 m
   (c) 0.02 m
   (d) 2 m
   (e) 0.002 m

   $dL = \frac{L}{N} = \frac{20\text{ cm}}{10} = 2\text{ cm} = 0.02\text{ m}$

3. Sketch the 10 pieces of the rod on the picture above. Label the pieces starting with piece #1 at the top of the rod and piece #10 at the bottom of the rod. Let's call the piece number n. What is the y-position of piece #3 ($n = 3$)?

   (a) $y = 0.05$ m
   (b) $y = 0.06$ m
   (c) $y = 0.04$ m
   (d) $y = 0.03$ m
   (e) $y = 0.1$ m

4. Which of the equations below can be used to calculate the y-position of any piece n? (Note: L is the length of the rod and dL is the length of a piece.

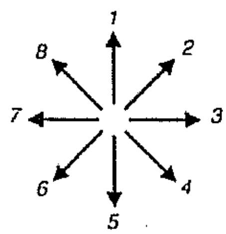
   (a) $y = \frac{L}{2} - \frac{n}{2}dL$
   (b) $y = \frac{L}{2} - \frac{(2n-1)}{2}dL$
   (c) $y = \frac{L}{2} - ndL$
   (d) $y = \frac{L}{2} - (2n-1)dL$
   (e) $y = \frac{L}{2} - (2n)dL$

   test for $n = 3$.

   $y = \frac{L}{2} - \frac{2(3)-1}{2}dL$
   $= 10\text{ cm} - \frac{5}{2}(2\text{ cm})$
   $= 10\text{ cm} - 5\text{ cm} = 5\text{ cm}$
   $= 0.05\text{ m}$ ✓

   Agrees with answer to #3

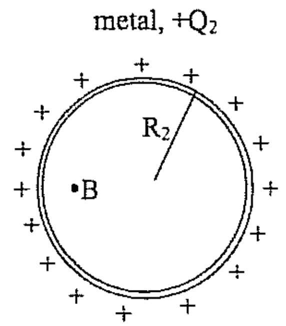
5. Which arrow points in the direction of $\vec{r}$ for calculating the electric field at point P due to piece #3?



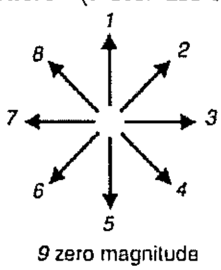
9 zero magnitude

   (a) 5
   (b) 8
   (c) 3
   (d) 4
   (e) 2

6. A metal sphere has a uniform positive charge as shown below.



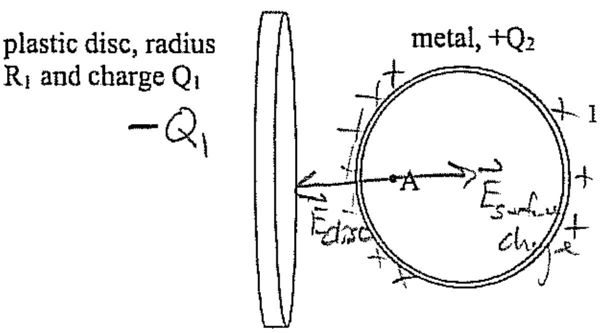
What is the direction of the electric field at point B due to charge on the surface of the sphere? (Note: use the direction arrows.)



uniform charge distribution

(a) 3
(b) 7
(c) 1
(d) 5
(e) 9, because $|\vec{E}_{net}|=0$

7. Suppose that the metal sphere in the previous question is brought near to a uniformly *negatively charged* disc as shown below.



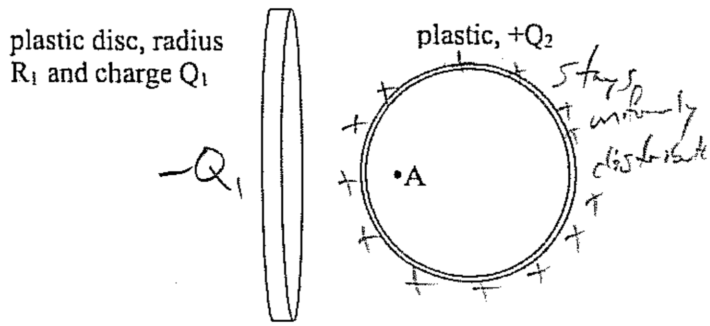
What is the direction of the *net* electric field at point A, if point A is along the axis of the disc?

(a) 8
(b) 9, because $|\vec{E}_{net}|=0$ the electrons are in static equilibrium
(c) 6
(d) 3
(e) 7

8. For the previous question, what is the direction of the electric field due to the charge on the surface of the sphere, at point A?

(a) 7
(b) 9, because $|\vec{E}_{net}|=0$
(c) 2
(d) 3
(e) 4

9. Now, suppose that you repeat the experiment with the uniformly charged sphere and disc, except this time you use a plastic sphere. Originally, the plastic sphere has a uniformly distributed positive charge on its surface. You bring it near to a uniformly negatively charged disc, as shown below.



What is the direction of the *net* electric field at point A?

$\vec{E}_{surface\ charge} = 0$

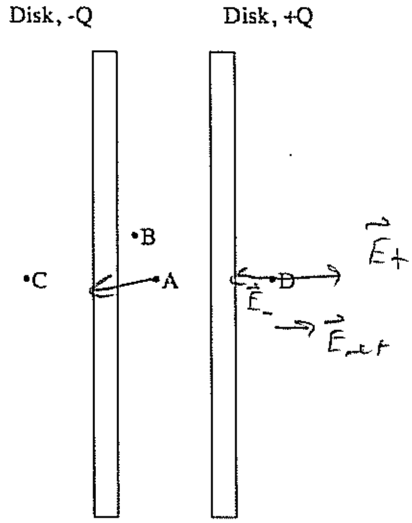
$\vec{E}_{net} = \vec{E}_{disc}$

(a) 7
(b) 9, because $|\vec{E}_{net}|=0$
(c) 3
(d) 8
(e) 6

10. For the previous question, what is the direction of the electric field due to the charge on the surface of the sphere, at point A?

(a) 3
(b) 7
(c) 9, because $|\vec{E}_{net}|=0$
(d) 2
(e) 4

A capacitor has plates (i.e. thin discs) of charge $\pm 0.1\mu C$. A side view is shown below. The radius of each plate is 50 cm and they are separated by a distance 5 mm. Points A, B, C, and D are all very close to the plates, relative to the size of the plates.



11. What is the direction of the electric field at point A?

(a) 9
(b) 3
(c) 1
(d) 7
(e) 5

12. What is the direction of the electric field at point D?

(a) 7
(b) 9
(c) 1
(d) 5
(e) 3

13. Rank the electric fields at points A, B, C, and D in order from *least* magnitude to *greatest magnitude.*

(a) $|\vec{E}_D| = |\vec{E}_C| = |\vec{E}_B| = |\vec{E}_A|$
(b) $|\vec{E}_D| = |\vec{E}_C| < |\vec{E}_B| = |\vec{E}_A|$
(c) $|\vec{E}_D| = |\vec{E}_C| < |\vec{E}_A| < |\vec{E}_B|$
(d) $|\vec{E}_D| = |\vec{E}_C| < |\vec{E}_B| < |\vec{E}_A|$
(e) $|\vec{E}_D| = |\vec{E}_C| = |\vec{E}_B| = |\vec{E}_A|$

14. What is the magnitude of the electric field at location A?

$E_{cap} = \frac{Q/A}{\epsilon_0}$

$= 14400 \frac{N}{C}$

(a) 7190 N/C
(b) 71.9 N/C
(c) 144 N/C
(d) 14400 N/C
(e) zero

15. If the separation of the discs of the capacitor is doubled, what happens to the magnitude of the electric field at points inside the capacitor? (Assume that even after doubling, the plate separation is still very small compared to the radius of the discs.)

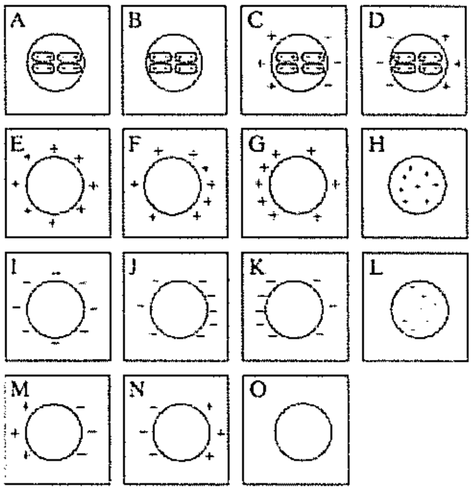
(a) It changes by a factor of 2.
(b) It changes by a factor of 4.
(c) It changes by a factor of 1/2.
(d) It changes by a factor of 1/4.
(e) It stays the same.

16. If you double the charge on the capacitor's discs (called plates), what happens to the magnitude of the electric field at points inside the capacitor?

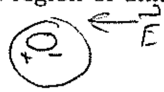
(a) It changes by a factor of 2.
(b) It changes by a factor of 4.
(c) It changes by a factor of 1/2.
(d) It changes by a factor of 1/4.
(e) It stays the same.

Section 2. Sketching charge distributions

For each situation described, sketch the charge distribution. Write the letter of the picture below most closely matches your sketch.



17. A neutral plastic sphere is in a region of uniform electric field in the $-x$ direction. Sketch the charge distribution of the sphere.



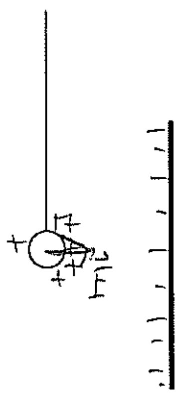
$\boxed{B}$ polarizes the neutral atoms

18. A neutral metal sphere is in a region of uniform electric field in the $-x$ direction. Sketch the charge distribution of the sphere.

$\boxed{M}$

polarizes sphere; sea of electrons shifts opposite $\vec{E}$.

19. A positively charged metal sphere is hanging from an insulating thread. You bring a negatively charged plastic rod toward the right side of the sphere, as shown below. Sketch the charge distribution of the sphere.

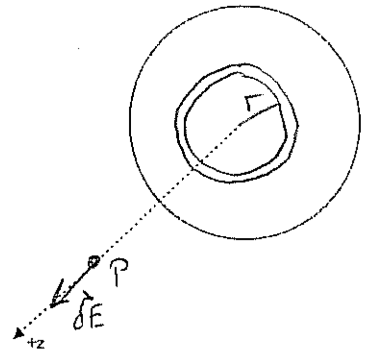


$\boxed{F}$

$\boxed{2 \text{ points}}$

Section 3. Problem Solving

20. A solid disc has a uniform surface charge Q and radius R. Derive an expression for the electric field at a point P on the axis of the disc. The steps below will help guide you through the derivation.



(a) Sketch a thin ring of the disc with charge dq, radius r, and width dr, and sketch $d\vec{E}$, the electric field at point P due to that ring.

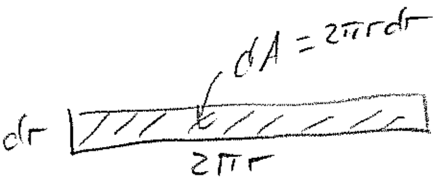
(b) Write an equation for $dE_{z,ring}$, the z-component of the electric field due to the thin ring, at point P. Be sure to use the radius of the ring r (not R which is the radius of the disc).

(c) Write the charge of the ring dq in terms of dr and substitute into $d\vec{E}$. (Note that if you cut the ring and straighten it out, its area is that of a rectangle, with length equal to the ring's circumference and width equal to dr.)

(d) Integrate (i.e. sum the contributions to the net electric field by all rings from $r = 0$ to $r = R$.) Express your final answer as a vector.

(b) $$dE_{z\,ring} = \frac{1}{4\pi\epsilon_0}\frac{dq\, z}{(r^2+z^2)^{3/2}}$$

(c) $$\frac{Q}{A} = \frac{dq}{dA_{ring}} = \frac{dq}{2\pi r\, dr}$$



ring layed out is a straight line

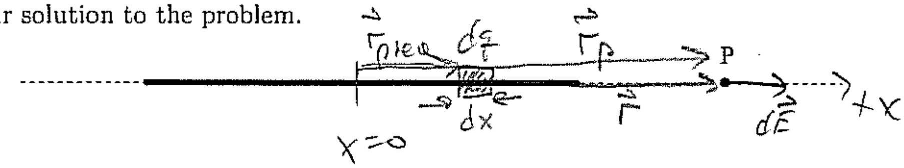
$$dE_{z\,ring} = \frac{1}{4\pi\epsilon_0}\frac{z}{(r^2+z^2)^{3/2}}\left(\frac{Q}{A}2\pi r\,dr\right)$$

$$= \frac{1}{4\pi\epsilon_0} Q/A\ 2\pi\ z\ \frac{r\,dr}{(r^2+z^2)^{3/2}}$$

(d) $$E_{disc_z} = \int_0^{r=R} dE_{ring_z} = \frac{(Q/A)z}{2\epsilon_0}\int_0^R \frac{r\,dr}{(r^2+z^2)^{3/2}} = \frac{(Q/A)z}{2\epsilon_0}\left(\frac{1}{z} - \frac{1}{(R^2+z^2)^{1/2}}\right)$$

$$\vec{E} = \left\langle 0, 0, \frac{(Q/A)}{2\epsilon_0}\left(1 - \frac{z}{(R^2+z^2)^{1/2}}\right)\right\rangle$$

A thin rod of length L has a uniform charge Q. Derive an expression for the electric field at point P on the axis of the rod that is at a distance d from the center of the rod. The questions below will help you organize your solution to the problem.



21. (a) Define your coordinate system, sketch a piece of the rod of charge dq, and sketch $d\vec{E}$, the electric field at point P due to that piece. Sketch position vectors for the piece and point P, and sketch the vector $\vec{r}$. Label everything correctly.

    (b) Write an equation for $d\vec{E}$, the electric field due to the piece of the rod at point P. Substitute $\vec{r}$.

    (c) Write the charge of the piece dq in terms of its length dx and substitute into $d\vec{E}$.

    (d) Integrate (i.e. sum the contributions to the net electric field by all pieces of the rod.) Express your final answer as a vector.

(b) $$dE_x = \frac{1}{4\pi\epsilon_0}\frac{dq}{|\vec{r}|^2}$$

$$\vec{r} = \vec{r}_P - \vec{r}_{piece} = \langle d,0,0\rangle - \langle x,0,0\rangle = \langle d-x,0,0\rangle$$

$$\hat{r} = \langle 1,0,0\rangle$$

$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{dq}{(d-x)^2}$$

(c) $$\frac{Q}{L} = \frac{dq}{dx} \quad \text{so} \quad dq = \frac{Q}{L}dx$$

$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{\frac{Q}{L}dx}{(d-x)^2}$$

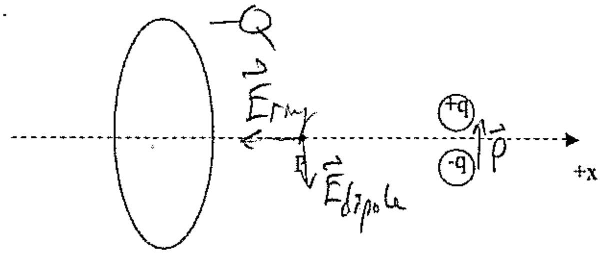
(d) $$E_{net_x} = \int_{x=-\frac{L}{2}}^{x=\frac{L}{2}} dE_x = \int_{x=-\frac{L}{2}}^{x=\frac{L}{2}} \frac{1}{4\pi\epsilon_0}\frac{Q/L}{(d-x)^2}dx$$

$$= \frac{1}{4\pi\epsilon_0} Q/L \underbrace{\left(\frac{1}{(d-\frac{L}{2})} - \frac{1}{(d+\frac{L}{2})}\right)}_{\frac{d+\frac{L}{2}-(d-\frac{L}{2})}{(d-\frac{L}{2})(d+\frac{L}{2})} = \frac{L}{d^2-(\frac{L}{2})^2}}$$

$$\boxed{E_{net_x} = \frac{1}{4\pi\epsilon_0}\frac{Q}{d^2-(\frac{L}{2})^2}}$$

$$\vec{E}_{net} = \left\langle \frac{1}{4\pi\epsilon_0}\frac{Q}{d^2-(\frac{L}{2})^2}, 0, 0 \right\rangle$$

22. A ring of radius 20 cm and uniform charge −10 nC, and a dipole of magnitude charge 1 μC and separation 0.01 mm, are configured as shown below.



What is the net electric field at point P which is 10 cm from the center of the ring and 10 cm from the center of the dipole? (Note: remember to give your answer as a vector and remember to check your answer with a sketch of what you expect for the direction of the net electric field.)

$$|\vec{E}|_{ring} = \frac{1}{4\pi\epsilon_0}\frac{|Q|z}{(R^2+z^2)^{3/2}} = \frac{1}{4\pi\epsilon_0}\frac{|(-10\times10^{-9}C)(0.1m)|}{((0.2m)^2+(0.1m)^2)^{3/2}}$$

$$= 805\ \frac{N}{C}$$

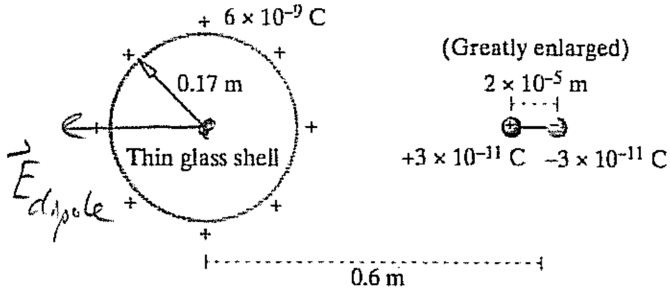
$$|\vec{E}|_{dipole} \approx \frac{1}{4\pi\epsilon_0}\frac{qs}{r^3} \quad \text{for } r \gg s$$

$$\approx \frac{1}{4\pi\epsilon_0}\frac{(1\times10^{-6})(0.01\times10^{-3})}{(0.1m)^3} = 90\ \frac{N}{C}$$

$$\vec{E}_{net} = \langle -805, -90, 0\rangle\ \frac{N}{C}$$

Note that $E_{net\,y}$ is small, so $\vec{E}_{net}$ is nearly horizontal. The ring basically dominates

23. A thin hollow spherical glass shell of radius 0.17 m carries a uniformly distributed positive charge 6×10^{-9} C as shown below. To the right of it is a horizontal permanent dipole with charges 3×10^{-11} C and -3×10^{-11} C separated by a distance 2×10^{-5} m (the dipole is shown greatly enlarged for clarity). The dipole is fixed in position and is not free to rotate. The distance from the center of the glass shell to the center of the dipole is 0.6 m.



(a) Calculate the net electric field at the center of the glass shell.

$\vec{E}_{net} = \vec{E}_{sphere} + \vec{E}_{dipole}$

0
Since charge is uniformly distributed

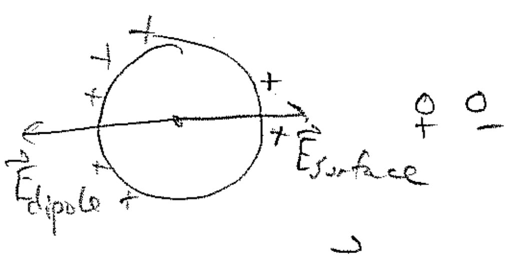
$|\vec{E}_{dipole}| \approx \frac{1}{4\pi\epsilon_0} \frac{2qs}{r^3}$ for $r >> s$

$\approx \left(9 \times 10^9 \frac{Nm^2}{C^2}\right)\left(\frac{2(3 \times 10^{-11} C)(2 \times 10^{-5} m)}{(0.6 m)^3}\right)$

$= 5 \times 10^{-5} \frac{N}{C}$

$\vec{E}_{net} = \langle -5 \times 10^{-5}, 0, 0 \rangle \frac{N}{C}$

(b) If the sphere were a solid metal ball with a charge 6×10^{-9} C, what would be the electric field due to the surface charge on the ball?



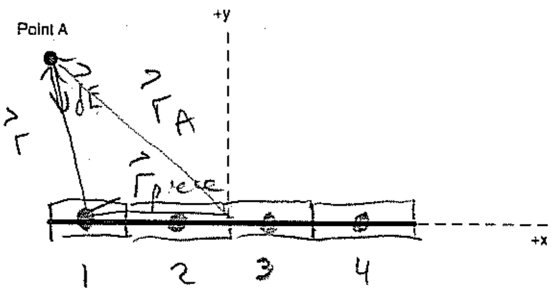
$\vec{E}_{net} = 0$ Since electrons are in static equilibrium

$\vec{E}_{surface} = -\vec{E}_{dipole}$

$= \langle 5 \times 10^{-5}, 0, 0 \rangle$

Q on sphere is irrelevant

24. A charged plastic rod has a uniform negative charge of $-5\ \mu C$ and a length 0.2 m. Point A is a point in space that is 0.05 m from the end of the rod at the location shown below. Numerically calculate the electric field at point A by breaking the rod into four equal length pieces and treating each piece as a point particle.



$dq = \frac{Q}{N} = \frac{(-5\times10^{-6}C)}{4} = -1.25\times10^{-6}C$

$dL = \frac{L}{N} = \frac{0.2m}{4} = 0.05m$

$\vec{r}_A = \langle -\frac{L}{2}, 0.05m, 0 \rangle$
$= \langle -0.1m, 0.05m, 0\rangle$

1: $\vec{r}_1 = \langle -\frac{3}{2}dL, 0, 0\rangle = \langle -0.075, 0, 0\rangle m$

$\vec{r} = \vec{r}_A - \vec{r}_1 = \langle -0.1m, 0.05m, 0\rangle - \langle -0.075, 0, 0\rangle m$

$= \langle -0.025, 0.05, 0\rangle m$ points left and upward, as expected.

$|\vec{r}| = 0.0559\ m$, $\hat{r}_1 = \langle -0.447, 0.894, 0\rangle$

$d\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{dq}{|\vec{r}_1|^2}\hat{r}_1 = \frac{1}{4\pi\epsilon_0}\frac{(-1.25\times10^{-6}C)}{(0.0559)^2}\langle -0.447, 0.894, 0\rangle$

$= \boxed{\langle 1.61\times10^{6}, -3.22\times10^{6}, 0\rangle \frac{N}{C}}$ rightward and downward, like the picture.

2: $\vec{r}_2 = \langle -\frac{1}{2}dL, 0, 0\rangle = \langle -0.025, 0, 0\rangle m$

$\vec{r} = \langle -0.075, 0.05, 0\rangle m$ $|\vec{r}| = 0.0901\ m$

$\hat{r} = \langle -0.832, 0.555, 0\rangle$

$\boxed{d\vec{E}_2 = \langle 1.15\times10^{6}, -7.69\times10^{5}, 0\rangle \frac{N}{C}}$

3: $\vec{r}_3 = \langle 0.025, 0, 0\rangle m$ $|\vec{r}| = 0.135\ m$ $\hat{r} = \langle -0.928, 0.371, 0\rangle$

$\vec{r} = \langle -0.125, 0.05, 0\rangle m$

$\boxed{d\vec{E}_3 = \langle +5.76\times10^{5}, -2.30\times10^{5}, 0\rangle \frac{N}{C}}$

4: $\vec{r}_4 = \langle 0.075, 0, 0\rangle m$, $\vec{r} = \langle -0.175, 0.05, 0\rangle m$, $|\vec{r}| = 0.182\ m$, $\hat{r} = \langle -0.962, 0.274, 0\rangle$

$\boxed{d\vec{E}_4 = \langle 3.27\times10^{5}, -9.31\times10^{4}, 0\rangle \frac{N}{C}}$

$\boxed{\vec{E}_{net} = \sum_{i=1}^{4}\vec{E}_i = \langle 3.66\times10^{6}, -4.31\times10^{6}, 0\rangle \frac{N}{C}}$

rightward and downward as expected!