Physics 222, Spring 2009

Quiz 1, Form: A

Name: Key

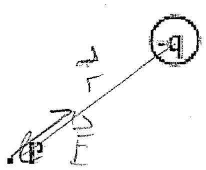
Date:

$$|\vec{E}|_{\text{dipole}} \approx \frac{1}{4\pi\epsilon_0}\frac{qs}{r^3} \quad \text{along perpendicular bisector if } r >> s$$

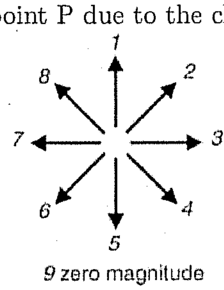
$$|\vec{E}|_{\text{dipole}} \approx \frac{1}{4\pi\epsilon_0}\frac{2qs}{r^3} \quad \text{along axis of dipole if } r >> s$$

Section 1. Multiple Choice

Questions 1–3: A negatively charged particle with charge $-q$ is at the location shown below.



1. What is the direction of the electric field at point P due to the charged particle?



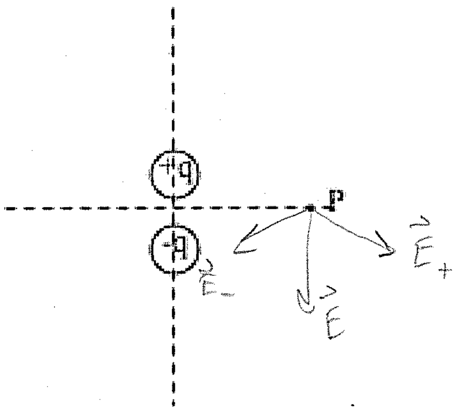
- (a) 4
- (b) 6
- (c) 1
- (d) 5
- (e) 2

2. If calculating the electric field at point P due to the charged particle, you must use the vector $\vec{r}$ which is the position of point P relative to the charged particle. When sketching $\vec{r}$, you should
   - (a) draw an arrow from point P to the charged particle.
   - (b) draw an arrow at point P that points in the direction of the electric field at point P.
   - (c) drawn an arrow at the charged particle that points in the direction of the electric field at point P.
   - (d) draw an arrow from the charged particle to point P.
   - (e) draw an arrow from the origin to point P.

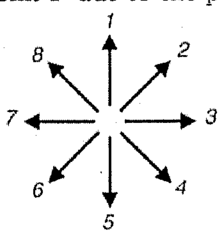
3. Suppose that the distance from the origin to point P in the previous question is r. If you double the distance from the particle to point P to $2r$, the electric field at point P will change by a factor

   $E \propto \frac{1}{r^2}$
   - (a) $\frac{1}{8}$
   - (b) $\frac{1}{2}$
   - (c) $\frac{1}{4}$
   - (d) 4
   - (e) None of the above, because the electric field due to a point particle is independent of distance.

Questions 4–6: A dipole is shown below.



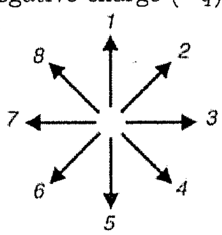
4. What is the direction of the electric field at point P due to the positive charge $(+q)$?



9 zero magnitude

(a) 8
(b) 1
(c) 4
(d) 5
(e) 3

5. Which arrow most closely points in the direction of the electric field at point P due to the negative charge $(-q)$?



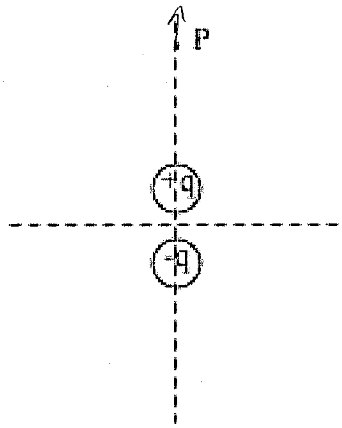
9 zero magnitude

(a) 5
(b) 6
(c) 2
(d) 1
(e) 7

6. Which arrow most closely points in is the direction of the net electric field at point P?

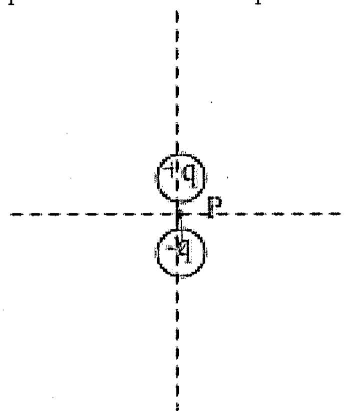
(a) None, because it is zero.
(b) 3
(c) 1
(d) 7
(e) 5

7. What is the direction of the electric field at point P due to the dipole shown below?



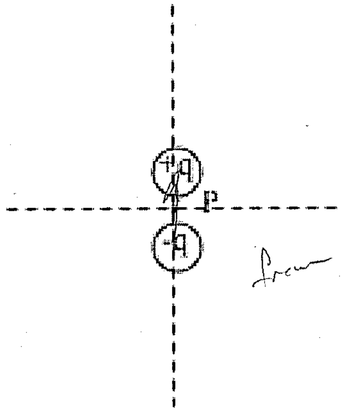
(a) 7
(b) 5
(c) 1
(d) 3
(e) None of the above, because it is zero.

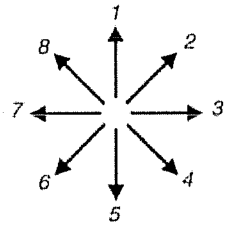
8. What is the direction of the electric field at point P due to the dipole shown below?



(a) None, because it is zero.
(b) 3
(c) 7
(d) 1
(e) 5

9. For the dipole shown below, what is the direction of the dipole moment vector?





9 zero magnitude

(a) 5
(b) 3
(c) 7
(d) 1
(e) None of the above.

10. At the location of a proton, there is an electric field (due to some other charged particle(s)) that is in the +x direction. What is the direction of the force on the proton by the electric field?

(a) in the +z direction.
(b) it is in all directions, radially away from the proton.
(c) in the –x direction.
(d) in the +x direction.
(e) None of the above.

11. Suppose that the electron in the previous question were replaced by a chlorine ion (Cl^-). The force on the chlorine ion would be

(a) greater than the force on a single electron at the same location.
(b) less than the force on a single electron at the same location.
(c) the same as the force on a single electron at the same location.

12. Suppose that the electron in the previous question were replaced by an alpha particle (He^{2+}). The force on the alpha particle would be

(a) greater than the force on a single electron at the same location, but opposite in direction.
(b) less than the force on a single electron at the same location, but opposite in direction.
(c) the same (in magnitude) as the force on a single electron at the same location, but opposite in direction.

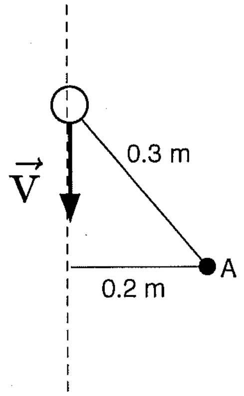
13. An electron has a velocity in the +x direction when it enters a region of uniform electric field that is in the –x direction. The electron will

(a) speed up.
(b) slow down.
(c) move with a constant velocity.
(d) be deflected upward.
(e) be deflected downward.

14. An electron has a velocity in the +x direction when it enters a region of uniform electric field that is in the +y direction. The electron will

(a) speed up.
(b) slow down.
(c) move with a constant velocity.
(d) be deflected upward.
(e) be deflected downward.

15. A particle of mass 0.1 kg has a speed of 2 m/s in the direction shown below. Its velocity is constant. What is its angular momentum about point A?



(a) $\langle 0, 0, 0.06\rangle$ kgm^2/s
(b) $\langle 0, 0, -0.06\rangle$ kgm^2/s
(c) $\langle 0, 0, 0.04\rangle$ kgm^2/s
(d) $\langle 0, 0, -0.04\rangle$ kgm^2/s
(e) zero

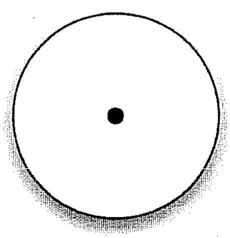
$\vec{L} = \vec{r} \times \vec{p}$

$|\vec{L}| = r_\perp |\vec{p}| = (0.2\,m)(0.1\,kg)(2\,\frac{m}{s})$

$= 0.04\ kg\,\frac{m^2}{s}$

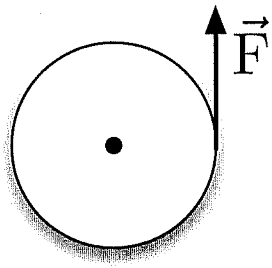
R.H. Rule: L_z is $+$.

16. The wheel shown below is rotating clockwise. Its axle is fixed. What is the direction of its angular momentum?



(a) +z direction
(b) -z direction
(c) Its angular momentum is zero.

17. For the wheel in the previous question, a force is applied in the direction shown below for a small time interval. What is the direction of the torque on the wheel due to this force?



(a) +z direction
(b) -z direction
(c) +y direction
(d) -y direction
(e) The torque is zero.

18. As a result of the applied torque, will the angular speed of the wheel increase or decrease?

(a) increase
(b) decrease
(c) neither, the angular speed of the wheel will be constant.

Questions 19–21: A baton twirler rotates a baton of length 0.7 m (radius 0.35 m) with a period of 0.25 s in a counterclockwise direction. She moves her arm so that the center of the baton travels at constant speed clockwise in a circle of radius 1.0 m, making one complete revolution in 3 second. The baton is made of a lightweight rod with 0.1 kg masses on each end.

19. Treat the baton as a point particle with all of its mass at its center of mass. What is the baton's translational angular momentum (about the center of the large circle)?

(a) $\langle 0, 0, 0.20\rangle$ kg m^2/s
(b) $\langle 0, 0, -0.20\rangle$ kg m^2/s
(c) $\langle 0, 0, -1.3\rangle$ kg m^2/s
(d) $\langle 0, 0, 0.42\rangle$ kg m^2/s
(e) $\langle 0, 0, -0.42\rangle$ kg m^2/s

clockwise

$\vec{L}_{trans} = \vec{r}_{CM} \times \vec{p}$

$|\vec{L}| = R_{circle}\, m V_{cm}$

$= R_{circle}\, m\, R\, \omega_{circle}$

$= 2(0.1\,kg)(1\,m)^2 * (\frac{2\pi}{3\,s})$

$= 0.42\,kg\,\frac{m^2}{s}$

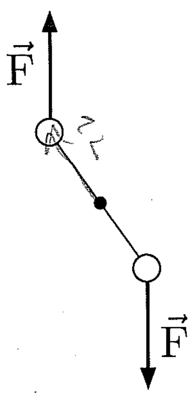
20. What is the baton's rotational angular momentum (about its center of mass)?

(a) $\langle 0, 0, -0.15\rangle$ kg m^2/s
(b) $\langle 0, 0, 0.15\rangle$ kg m^2/s
(c) $\langle 0, 0, 0.62\rangle$ kg m^2/s
(d) $\langle 0, 0, -0.62\rangle$ kg m^2/s
(e) $\langle 0, 0, 0.023\rangle$ kg m^2/s

21. What is the baton's total angular momentum?

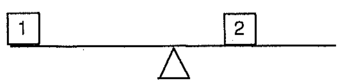
(a) $\vec{L}_{total} = \vec{L}_{trans} + \vec{L}_{rot}$
(b) $\vec{L}_{total} = \vec{r}_{CM} \times \vec{p}_{total}$
(c) $\vec{L}_{total} = \vec{L}_{trans} - \vec{L}_{rot}$
(d) $\vec{L}_{total} = \vec{L}_{trans}$
(e) $\vec{L}_{total} = \vec{L}_{rot}$

22. A diatomic molecule is shown below as if it is a barbell. An electric field exerts a force upward on one particle and downward on the other particle. The net torque on the molecule about its center of mass is



(a) zero.
(b) in the $+z$ direction.
(c) in the $-z$ direction.

Questions 23–24: A lightweight level beam of uniform density has two objects on top of the beam. Object 1 has a mass of 0.1 kg and is at a distance of 0.5 m from the center of the beam. Object 2 has a mass of 0.15 kg and is at a distance of 0.2 m from the beam. The beam is free to rotate about the fulcrum (i.e. pivot).



23. At this instant, what is the net torque on the beam?

(a) $\vec{\tau}_{net} = \langle 0, 0, 0.49\rangle$ N m
(b) $\vec{\tau}_{net} = \langle 0, 0, -0.294\rangle$ N m
(c) $\vec{\tau}_{net} = \langle 0, 0, 0.784\rangle$ N m
(d) $\vec{\tau}_{net} = \langle 0, 0, -0.784\rangle$ N m
(e) $\vec{\tau}_{net} = \langle 0, 0, 0.196\rangle$ N m

24. Where would you place a third object of mass 0.05 kg in order for the beam to be in equilibrium?

(a) 0.4 m to the left of the fulcrum
(b) 0.3 m to the left of the fulcrum
(c) 0.3 m to the right of the fulcrum
(d) 0.4 m to the right of the fulcrum
(e) None of the above. Regardless of where you place this mass on the beam, the system will not be balanced.

25. Suppose that an electron in a hydrogen atom is in the state n=2. What are the possible values of the magnitude of the electron's orbital angular momentum?

(a) $L = 0,\ L = \sqrt{2}\hbar,\ L = \sqrt{6}\hbar$
(b) $L = 0,\ L = \sqrt{6}\hbar$
(c) $L = 0,\ L = 1\hbar,\ L = 2\hbar$
(d) $L = 0,\ L = 2\hbar$
(e) $L = 0,\ L = \sqrt{2}\hbar$

26. If an electron in hydrogen has an orbital angular momentum quantum number $l = 2$, how many different orientations of the orbital angular momentum are possible? (It helps to list them or sketch them and then count how many there are.)

(a) 5
(b) 4
(c) 3
(d) 2
(e) 1

27. A neutral chlorine atom has 17 electrons. At low temperatures, all of the electrons are in their lowest possible energy states. (In other words, none of them are excited into higher energy orbitals and the lowest energy orbitals are as filled, as much as possible.) How many electrons have the quantum numbers $n = 3$ and $l = 2$? To answer this question, you should sketch an orbital diagram.

(a) 7
(b) 6
(c) 0
(d) 5
(e) 3

28. What condition is required for the linear momentum of a system to be constant?

(a) $\vec{F}_{net} = 0$
(b) $\vec{\tau}_{net} = 0$
(c) E must be constant.

29. What condition is required for the angular momentum of a system to be constant?

(a) $\vec{F}_{net} = 0$
(b) $\vec{\tau}_{net} = 0$
(c) E must be constant.

$n=1, l=0, m_l=0, m_s=\pm\frac{1}{2}$ } $2e^-$

1s ↑↓

$n=2, l=0, m_l=0, m_s=\pm\frac{1}{2}$ } $2e^-$

2s ↑↓

$n=2, l=1, m_l=-1, m_s=\pm\frac{1}{2}$
$0, \pm\frac{1}{2}$
$+1, \pm\frac{1}{2}$ } $6e^-$

2p ↑↓ ↑↓ ↑↓

3s ↑↓ $n=3, l=0, m_l=0, m_s=\pm\frac{1}{2}$ } $2e^-$

$l=1, m_l=-1, m_s=\pm\frac{1}{2}$
$0, m_s=\pm\frac{1}{2}$
$+1, m_s=\pm\frac{1}{2}$ } 6 possible states only 5 filled for Cl

3p ↑↓ ↑↓ ↑

None have $l=2$! These orbitals are empty!

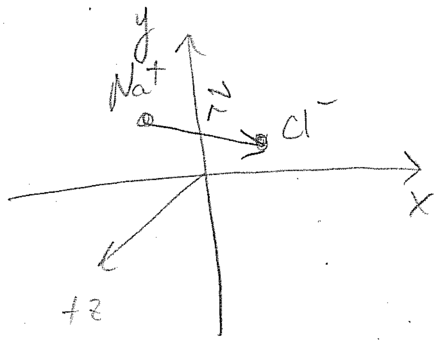
Note: I answered #31 first!
You can solve it eitherway.

Section 2. Problem Solving

For the questions below, you must sketch all vectors, including $\vec{r}$ and $\vec{E}$ and $\vec{F}$, as appropriate. You must also correctly add vectors when using the Principle of Superposition.

30. A Na^+ ion is at the location $\langle -2, 3, 0\rangle$ nm, and a Cl^- ion is at the location $\langle 1, 1, 1\rangle$ nm. What is the force on the sodium ion? (Note: your final answer must be expressed as a vector.)

31. What is the force on the chlorine ion? (No calculation is needed.)



$\vec{E}$ at Cl^- due to Na^+ is:...

$\vec{r} = \vec{r}_{Cl^-} - \vec{r}_{Na^+}$

$= \langle 1,1,1\rangle - \langle -2,3,0\rangle$ nm

$= \langle 3,-2,1\rangle \times 10^{-9}$ m

$|\vec{r}| = 3.74\times10^{-9}$ m

$\hat{r} = \langle 0.802, -0.534, 0.267\rangle$

$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q_{Na^+}}{|\vec{r}|^2}\hat{r}$ at location of Cl^-

$\approx \left(9\times10^9 \frac{Nm^2}{C^2}\right)\left(\frac{1.6\times10^{-19}C}{(3.74\times10^{-9}m)^2}\right)\hat{r}$

$= 1.03\times10^8 \frac{N}{C}\hat{r}$

$= \langle 8.26\times10^7, -5.50\times10^7, 2.75\times10^7\rangle \frac{N}{C}$

$\vec{F}_{on\ Cl^-} = q_{Cl^-}\vec{E}_{due\ to\ Na^+}$

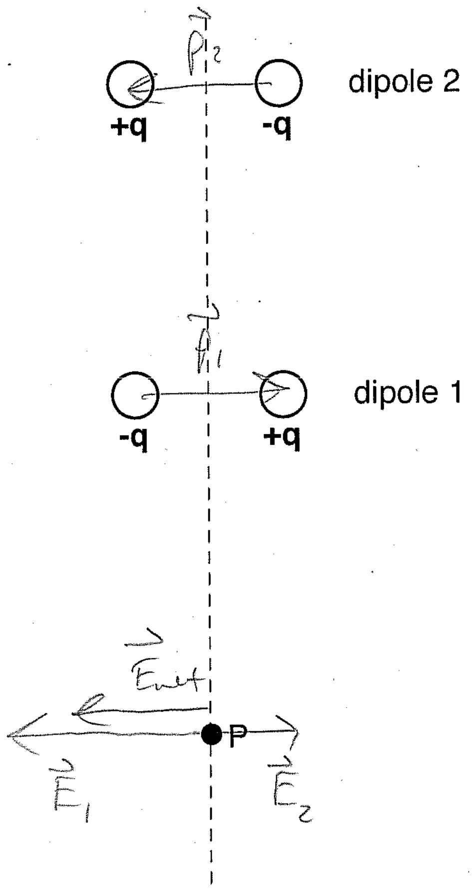
$= (-1.6\times10^{-19}C)\vec{E}$

$\boxed{\vec{F}_{on\ Cl^-} = \langle -1.3\times10^{-11}, 8.8\times10^{-12}, -4.4\times10^{-12}\rangle N}$

Note that $\vec{F}$ is opposite $\vec{E}$ as expected for Cl^-.

$\boxed{\vec{F}_{on\ Na^+} = -\vec{F}_{on\ Cl^-} = \langle 1.3\times10^{-11}, -8.8\times10^{-12}, 4.4\times10^{-12}\rangle N}$

32. Two dipoles are oriented as shown below. The dipoles are composed of charges $+e$ and $-e$ that are separated by a distance 1×10^{-10} m. Dipole 1 is a distance 1×10^{-8} m from point P. Dipole 2 is twice distance the distance from point P as Dipole 1 (i.e. it is 2×10^{-8} m from point P). In this case, you may use the approximation that $r >> s$. What is the net electric field at point P? You must express your answer as a vector, you must sketch the electric field due to each dipole, and you must sketch the net electric field. You must include correct units.



$$|\vec{E}_1| = \frac{1}{4\pi\epsilon_0}\frac{qs}{r_1^3}$$

$$= \left(9 \times 10^9 \frac{Nm^2}{C^2}\right)\left(\frac{(1.6 \times 10^{-19} C)(1 \times 10^{-10} m)}{(1 \times 10^{-8} m)^3}\right)$$

$$= 1.44 \times 10^5 \frac{N}{C}$$

$$|\vec{E}_2| = \frac{1}{4\pi\epsilon_0}\frac{qs}{r_2^3}$$

$$= \frac{1}{8} E_1 \quad \text{since } r_2 = 2r_1 \text{ and } r_2^3 = 8r_1^3$$

$$= 1.8 \times 10^4 \frac{N}{C}$$

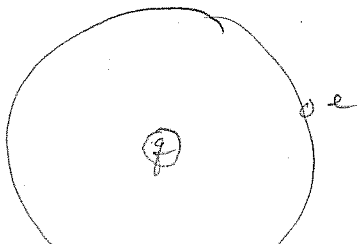
$|\vec{E}_1| > |\vec{E}_2|$ so E_{net} is in direction of $\vec{E}_1$

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_{net} = \langle -1.44 \times 10^5 \frac{N}{C}, 0, 0\rangle + \langle 1.8 \times 10^4, 0, 0\rangle \frac{N}{C}$$

$$= \langle -1.26 \times 10^5, 0, 0\rangle \frac{N}{C}$$

33. Use the Bohr model to derive an expression for the radius of single-electron atom with Z protons in the nucleus. Substitute values for $\hbar$, $m_{electron}$, e, etc. to write your expression in terms of the Bohr radius (i.e. the radius of hydrogen in its ground state). What is the ratio of the radius of hydrogen in its ground state to the radius of singly ionized helium (i.e. He^{1+}) in its ground state?



$q_{nucleus} = +Ze$

Newt. 2nd law: $|\vec{F}_{net}| = \frac{mv^2}{r} = \frac{p^2}{mr}$

Bohr model: $L = n\hbar$, $n = 1, 2 \ldots$

$rp = n\hbar$

$p = \frac{n\hbar}{r}$

Coulomb's law gives F on electron:

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{(Ze)e}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2}$$

Thus, Newt 2nd law gives:

$$\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2} = \frac{p^2}{mr}$$

$$\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^{\not 2}} = \frac{n^2\hbar^2}{mr^{\not 3}}$$

$$r = \frac{n^2}{Z}\left(\frac{\hbar^2}{\frac{1}{4\pi\epsilon_0}me^2}\right)$$

$$= \frac{n^2}{Z}\left(\frac{(1.055\times10^{-34}\,J\cdot s)^2}{(8.987\times10^{9})(9.109\times10^{-31}\,kg)(1.602\times10^{-19}\,C)^2}\right)$$

$$= \frac{n^2}{Z}\left(5.30\times10^{-11}\,m\right) = \frac{n^2}{Z}\underbrace{\left(0.530\times10^{-10}\,m\right)}_{\text{Bohr radius}}$$

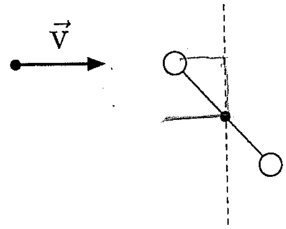
$\frac{r_H}{r_{He^+}} = 2$ since $Z_{He} = 2Z_H$; He has 2 protons

34. A diatomic molecule of nitrogen can be modeled as a rigid rotor (i.e. barbell) as shown below. Nitrogen has a mass of about 14 g/mol and the length of the bond is about 1×10^{-12} m (so its radius is half this length).

$R = 0.5 \times 10^{-12}$ m

$M_N = \left(14 \frac{g}{mol}\right)\left(\frac{1 kg}{1000 g}\right)\left(\frac{1 mol}{6 \times 10^{23} atoms}\right) = 2.33 \times 10^{-26}$ kg

$m_{Li} = \left(7 \frac{g}{mol}\right)\left(\frac{1 kg}{1000 g}\right)\left(\frac{1 mol}{6 \times 10^{23} atoms}\right) = 1.17 \times 10^{-26}$ kg.



An incoming lithium atom (mass 7 g/mol) with a speed of 1×10^5 m/s collides with a nitrogen nucleus and *reflects backwards in the* $-x$ *direction* with a speed of 6×10^4 m/s. As a classical model, let's assume that the diatomic molecule is initially not rotating (which is not possible of course).

(a) What is the initial angular momentum of the system with respect to the center of the nitrogen molecule?

(b) What is the final angular momentum of the system with respect to the center of the nitrogen molecule?

(c) What is the final angular momentum of the lithium nucleus?

(d) How fast will the molecule rotate after the collision?

(a) $\vec{L}_i = \vec{L}_{Li} = \vec{r} \times \vec{p}$

$|\vec{L}| = |r_\perp||m\vec{v}| = R\sin(45°) m_{Li} v_i$

$L_{iz} = -R\sin(45°) m_{Li} v_i$

$= -(0.5 \times 10^{-12} m)(1.17 \times 10^{-26} kg)(1 \times 10^5 \frac{m}{s}) \sin(45°)$

$= -4.14 \times 10^{-34}$ kg $\cdot \frac{m^2}{s}$

(b) $\vec{L}_f = \vec{L}_i$ so $L_{fz} = -4.14 \times 10^{-34}$ kg $\cdot \frac{m^2}{s}$

(c) $L_{fLi} = +R\sin(45°) m_{Li} v_f$

$= (0.5 \times 10^{-12} m)(1.17 \times 10^{-26} kg)(6 \times 10^4 \frac{m}{s}) \sin(45°)$

$= 2.48 \times 10^{-34}$ kg $\cdot \frac{m^2}{s}$

(d) $\vec{L}_f = \vec{L}_{fLi} + \vec{L}_{fN_2}$

so $L_{fN_2z} = L_{fz} - L_{fLiz} = (-4.14 \times 10^{-34} - 2.48 \times 10^{-34})$ kg $\cdot \frac{m^2}{s}$

$= -6.62 \times 10^{-34}$ kg $\cdot \frac{m^2}{s}$

$L_z = I\omega_z$

so $\omega_z = \frac{L_z}{I} = \frac{L_z}{(2mR^2)} = \frac{-6.62 \times 10^{-34} kg \cdot \frac{m^2}{s}}{2(2.33 \times 10^{-26} kg)(0.5 \times 10^{-12} m)^2}$

Answer Key for Exam A

Section 1. Multiple Choice

1. (e)
2. (d)
3. (c)
4. (c)
5. (b)
6. (e)
7. (c)
8. (e)
9. (d)
10. (c)
11. (c)
12. (a)
13. (a)
14. (e)
15. (c)
16. (b)
17. (a)
18. (b)
19. (e)
20. (c)
21. (a)
22. (c)
23. (e)
24. (d)
25. (e)
26. (a)
27. (c)
28. (a)
29. (b)