Physics 221, Fall 2009

Quiz 5, Form: $\boxed{A}$

Name: Key

Date: ____________________

$G = 6.7 \times 10^{-11}$ N m^2/kg^2

$M_{Sun} = 2 \times 10^{30}$ kg

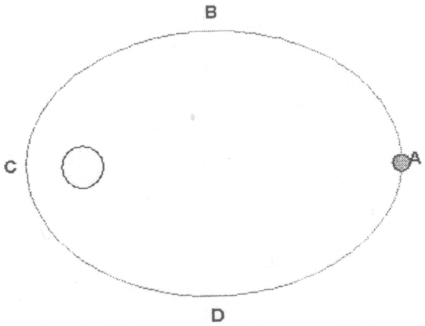
$M_{Earth} = 6.4 \times 10^{24}$ kg

$R_{Earth} = 6.4 \times 10^{6}$ m

$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 N\ m^2/C^2$

Section 1. Multiple Choice

Questions 1–4: A comet orbits the Sun as shown below.



1. At which point is the gravitational potential energy of the system (of comet and Sun) a minimum?

   (a) A
   (b) B
   (c) C
   (d) D
   (e) None of the above because it is constant.

2. At the point in the orbit where the gravitational potential energy of the system is a minimum, the kinetic energy of the system is

   (a) a minimum.
   (b) a maximum.
   (c) the same as at all other points (because it is constant).
   (d) zero.
   (e) negative.

3. The total energy of the system (of comet and Sun) is

   (a) positive. It is bound.
   (b) negative.
   (c) zero.

Questions 4–6: A person uses a rope to pull a 100-kg boat. She pulls with a constant force $< 4, -2, 0 >$ N through a displacement of $< 3, -3, 0 >$ m. The boat speeds up from 0.5 m/s to 1.0 m/s.

4. What is the change in the energy of the boat?

   (a) 37.5 J
   (b) 55.5 J
   (c) 19.5 J
   (d) 18.0 J
   (e) 57 J

   $\Delta E = \Delta K$
   $= \frac{1}{2} m (v_f^2 - v_i^2)$
   $= 37.5 J$

5. What is the work done by the person?

   (a) 37.5 J
   (b) 55.5 J
   (c) 19.5 J
   (d) 18.0 J
   (e) zero

   $W = \vec{F} \cdot \Delta \vec{r}$
   $= F_x \Delta x + F_y \Delta y + F_z \Delta z$
   $= 4(3) + (-2)(-3)$
   $= 12 + 6 = 18 J$

6. The only other force that does work is the water. What is the work done by the water on the boat?

   (a) 37.5 J
   (b) 55.5 J
   (c) 19.5 J
   (d) 18.0 J
   (e) zero

   $W_1 + W_2 = \Delta K$
   $W_2 = \Delta K - W_1$
   $= 37.5 J - 18 J$
   $= 19.5 J$

7. If the total work done on a particle is negative, then the particle

   (a) will have a constant speed.
   (b) will speed up.
   (c) will slow down.

   $W = \Delta E$
   $W = \Delta K$
   ΔK is $-$

8. Suppose that a potential energy function for a particular interaction of two particles is given by $U = -A/x^2$. What is the x-component of the force on a particle?

(a) $F_x = \frac{-2A}{x}$

(b) $F_x = \frac{-A}{2x}$

(c) $F_x = \frac{-2A}{x^2}$

(d) $F_x = \frac{-3A}{x^3}$

(e) $F_x = \frac{-2A}{x^3}$

$F_x = -\frac{dU}{dx}$

$= -\frac{d}{dx}(-Ax^{-2})$

$= -2Ax^{-3}$

$= \frac{-2A}{x^3}$

9. You stretch a spring of stiffness 40 N/m from $s = 0.05$ m to $s = 0.10$ m. How much work did you do on the spring?

(a) zero

(b) 0.2 J

(c) 0.05 J

(d) 0.25 J

(e) 0.15 J

$W = \int F_x dx$

$= \int_{0.05}^{0.1} kx\,dx$

$= \frac{1}{2}kx^2 \Big|_{0.05}^{0.1}$

$= \frac{1}{2}k\{(0.1)^2 - 0.05^2\} = 0.15\text{ J}$

10. As the distance between two massive bodies increases, the gravitational potential energy of the system

(a) increases.

(b) decreases.

(c) remains constant.

U increases with distance

11. A roller coaster is almost at rest at the top of an 80-m tall hill (i.e. 80 m above the ground). The bottom of the hill is 10 m above the ground. If friction is negligible, what will be the speed of the roller coaster at the bottom of the hill?

(a) 14 m/s

(b) 19 m/s

(c) 40 m/s

(d) 26 m/s

(e) 37 m/s

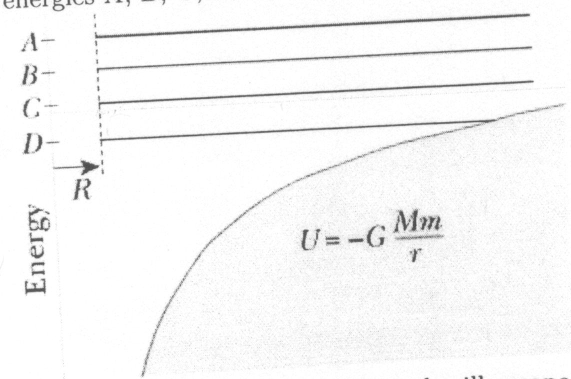
$y_i = 80\text{m}$

$y_f = 10\text{m}$

$mgy_i + \cancel{\frac{1}{2}mv_i^2} = mgy_f + \frac{1}{2}mv_f^2$

$v_f = \sqrt{2g(y_i - y_f)} = 37\frac{m}{s}$

12. Four objects are launched from a planet with energies A, B, C, and D.



Which objects are "unbound" and will escape the planet (i.e. have "open" orbits)?

(a) A

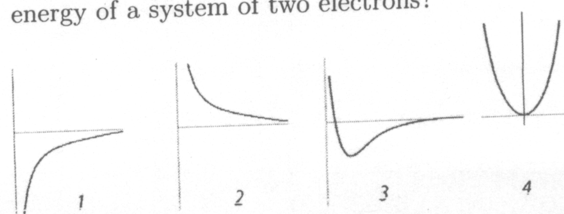
(b) A, B, and C

(c) A, B, C, and D

(d) D

(e) None of the above

13. Which of the graphs below shows the potential energy of a system of two electrons?



(a) 1

(b) 2

(c) 3

(d) 4

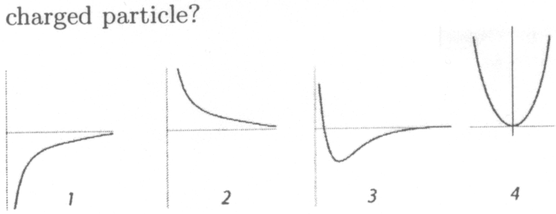
(e) None of the above

U is pos. sinc

$U = (9\times10^9)\frac{q_1 q_2}{r}$

and $q_1 q_2$ is +

14. Which of the graphs below shows the potential energy of a system composed of an interacting positively charged particle and a negatively charged particle?



(a) 1
(b) 2
(c) 3
(d) 4
(e) None of the above

U is − since $f_1 f_2$ is −.

15. The total energy of an Earth-rocket system is −9500 J. The minimum energy needed to make the rocket escape Earth's orbit is

(a) 9500 J
(b) less than 9500 J
(c) greater than 9500 J

E = 0 to escape.
$W = \Delta E = 0 - (-9500\text{J}) = 9500\text{J}$

16. What is the escape speed for a rocket launched from the surface of Earth?

(a) 7900 m/s
(b) 6.3×10^7 m/s
(c) 5600 m/s
(d) 1.1×10^4 m/s
(e) 465 m/s

$V = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.7\times10^{-11})(M_E)}{R_E}} = 1.16\times10^4 \frac{m}{s}$

17. A black hole is an object that has such a large mass and small radius that the escape speed is greater than the speed of light. Suppose that a black hole has a mass of 5 times the mass of our Sun ($M_{black\ hole} = 1 \times 10^{31}$) kg). At what distance R from the black hole is the escape speed equal to the speed of light? (This distance is called the event horizon for the black hole.)

(a) 3 km
(b) 5 km
(c) 1 km
(d) 10 km
(e) 15 km

$V = \sqrt{\frac{2GM}{R}} = c$

$\frac{2GM}{R} = c^2$

$R = \frac{2GM}{c^2}$

$R = \frac{2(6.7\times10^{-11})(1\times10^{31})}{(3\times10^8)^2} = 1.5\times10^4\ m = 15\ Km$

18. What initial speed is required to toss a water balloon vertically a height of 10 m above your hand?

(a) 10 m/s
(b) 7 m/s
(c) 14 m/s
(d) 3 m/s
(e) 17 m/s

$v_f = 0$ $y_f = ?$
v_i $y_i = 0$
$\cancel{m}gy_f = \frac{1}{2}\cancel{m}v_i^2$
$v_i = \sqrt{2gy_f} = 14 \frac{m}{s}$

19. A roller coaster begins at rest on the top of a hill. The total energy of the roller coaster and Earth is E, and its initial potential energy relative to the bottom of the hill is U. Treat the roller coaster as a frictionless roller coaster. After the roller coaster drops 1/2 of its initial height, its kinetic energy is

(a) $1/2U_i$
(b) $1/4U_i$
(c) $3/4U_i$
(d) U_i
(e) $1/\sqrt{2}U_i$

$E = U_i$
$U = \frac{1}{2}U_i,\ K = \frac{1}{2}U_i$
$E = K_f$
Since $U = mgy$

20. For the roller coaster in the previous question, when it has dropped half of its initial height, its speed will be

$K = \frac{1}{2}K_f$ so $v^2 = \frac{1}{2}v_f^2$ and $v = \frac{1}{\sqrt{2}}v_f$

(a) 1/4 of its speed at the bottom of the hill.
(b) 1/2 of its speed at the bottom of the hill.
(c) $1/\sqrt{2}$ of its speed at the bottom of the hill.
(d) 3/4 of its speed at the bottom of the hill.
(e) equal to its speed at the bottom of the hill.

21. You pull your little sister across a flat snowy field on a sled. Your sister plus the sled have a mass of 22 kg. The rope is at an angle of 33 degrees to the ground. As you pull with a force of 33 N, the sled travels a distance of 53 m.

What is the work done by you on your sister?

(a) 1.14×10^4 J
(b) 9580 J
(c) 1750 J
(d) 1470 J
(e) 950 J

$W = \vec{F} \cdot \vec{\Delta r}$
$= |\vec{F}||\vec{\Delta r}|\cos\theta$
$= (33N)(53m)\cos(33)$
$= 1500\text{J}$

$\vec{F}$ θ $\vec{\Delta r}$

Section 2. Problem Solving

22. A proton and an antiproton traveling toward each other with a speed of $0.9c$ collide and annihilate each other, producing two photons. What is the kinetic energy of each photon after the annihilation? The mass of a proton is the same as the mass of an antiproton, 1.67×10^{-27} kg. Neglect the change in electric potential energy of the system because it is negligible compared to the change in rest energy of the system.

$\boxed{i}$

p^+ $\qquad\qquad$ p^-

$E_{1i} = ?$

$v_{1i} = 0.9c$

$v_{2i} = 0.9c$

$E_{2i} = ?$

$$E_{p^+} = \gamma mc^2 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} mc^2 = \frac{1}{\sqrt{1-0.9^2}}(1.67\times10^{-27})(3\times10^8)^2 = 3.45\times10^{-10}\,J$$

$E_{p^-} = 3.45\times10^{-10}\,J$

$\boxed{f}$

photon$_1$ $\qquad$ photon$_2$

$K_{photon_1} = ?$

$K_{photon_2} = ?$

closed system so $W = \Delta E = 0$

$$\Delta E = 0$$

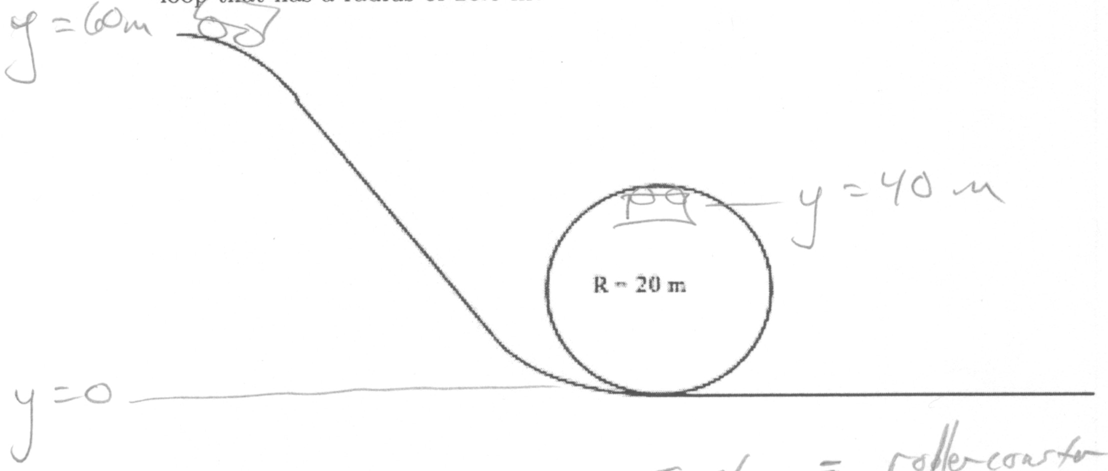
$$E_i = E_f$$

$$E_{p1} + E_{p2} = E_{photon_1} + E_{photon_2}$$

$$2E_p = 2E_{photon}$$

$$E_{photon} = E_p = \boxed{3.45\times10^{-10}\,J}$$

23. A roller coaster has a speed of 2.0 m/s at the top of a 60.0-m tall hill (relative to the bottom of the track) as shown below. If friction is negligible, what is the speed of the roller coaster at the top of the loop that has a radius of 20.0 m?



$y_i = 60\,m$

$y_f = 40\,m$

$v_i = 2\,\frac{m}{s}$

$v_f = ?$

System ≡ roller coaster + Earth

closed system so $\Delta E = 0$

$$E_i = E_f$$

$$mgy_i + \frac{1}{2}mv_i^2 = mgy_f + \frac{1}{2}mv_f^2$$

$$\frac{1}{2}\cancel{m}v_f^2 = \cancel{m}g(y_i - y_f) + \frac{1}{2}\cancel{m}v_i^2$$

$$\frac{1}{2}v_f^2 = g(y_i - y_f) + \frac{1}{2}v_i^2$$

$$v_f = \left(2\left[g(y_i - y_f) + \frac{1}{2}v_i^2\right]\right)^{1/2}$$

$$= \left(2\left[(9.8)(20) + \frac{1}{2}(2)^2\right]\right)^{1/2}$$

$$\boxed{v_f \approx 20\,\frac{m}{s}}$$

24. A hydrogen nucleus (1 proton) and a helium nucleus (2 protons) that start out very far apart travel straight toward each other and collide, each with an initial kinetic energy of 0.15 MeV. What is the closet distance between them when they momentarily come to a stop during the collision? (Note: the radius of a proton is approximately 1×10^{-15} m. Do the nuclei get closer than this distance? If they do, then they would have enough energy to perhaps break apart the helium nucleus or break apart the protons, which are made of bound quarks.)

i $\qquad$ f $\qquad$ i

H $\qquad$ H He $\qquad$ He

$q_H = 1.6 \times 10^{-19}\,C$

$v_f = 0$ so

$K_f = 0$

$q_{He} = 2(1.6 \times 10^{-19}\,C) = 3.2 \times 10^{-19}\,C$

system = H + He atoms

closed system so $W = \Delta E = 0$

so $E_i = E_f$

$U_{elec_i} + K_i = U_{elec_f} + K_f$ (no charge in rest energy)

$$\cancelto{0}{\frac{(9\times10^9) q_1 q_2}{r_i}} + K_{H_i} + K_{He_i} = \frac{9\times10^9 q_1 q_2}{r_f} + \cancelto{0}{K_{f_H}} + \cancelto{0}{K_{f_{He}}}$$

$$K_H = (0.15 \times 10^6\,eV)\left(1.6 \times 10^{-19}\,\frac{J}{eV}\right)$$

$$= 2.4 \times 10^{-14}\,J$$

$$2(2.4 \times 10^{-14}\,J) = \frac{(9\times10^9)(1.6\times10^{-19}\,C)(3.2\times10^{-19}\,C)}{r_f}$$

$$\boxed{r_f = 9.6 \times 10^{-15}\,m}$$

Section 3. LAB

25. An electron has mass 9×10^{-31} kg. If the electron's speed is $0.975c$, what is its total energy, in joules?

$$E = \gamma mc^2 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\, mc^2 = \frac{1}{\sqrt{1-(0.975)^2}}\,(9\times10^{-31})(3\times10^{8})^2$$
$$= \boxed{3.65\times10^{-13}\ \text{J}}$$

26. For the electron in the previous question, what is its rest energy?

$$E_{rest} = mc^2$$
$$= (9\times10^{-31}\,\text{kg})(3\times10^{8})^2 = \boxed{8.1\times10^{-14}\ \text{J}}$$

27. For the electron in the previous question, what is its kinetic energy?

$$E = E_{rest} + K$$
$$K = E - E_{rest} = 3.65\times10^{-13}\,\text{J} - 8.1\times10^{-14}\,\text{J} = \boxed{2.84\times10^{-13}\ \text{J}}$$

28. Why can a particle with mass NEVER reach the speed of light? (Even if you exert a force on the particle and accelerate it for a very long time.)

$E = \gamma mc^2$ If $v = c$, then $\gamma = \frac{1}{0} = \infty$ and $E = \infty$.

It would have infinite energy and would require infinite work to accelerate it to this energy.

29. A 0.5 kg cart travels down a track. It starts from rest at a height of 0.5 m above the table. You select a point on the track that is 0.2 m above the table to measure the speed of the cart and find it to be 2.3 m/s. Define the system to be the cart.

(a) What is the work done by the gravitational force by Earth on the cart?

$$W = \vec{F}_{grav} \cdot \Delta\vec{r} = F_y\,\Delta y = -mg(y_f - y_i)$$
$$= -(0.5)(9.8)(0.2 - 0.5) = \boxed{1.47\ \text{J}}$$

(b) What is the change in the kinetic energy of the cart?

$$\Delta K = \frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2}^{\,0}$$
$$= \frac{1}{2}(0.5)(2.3)^2 = \boxed{1.32\ \text{J}}$$

(c) How much energy is "lost" due to friction?

$W_{grav} + W_{fric} = \Delta K$ so $W_{fric} = 1.32\,\text{J} - 1.47\,\text{J}$

$$= \boxed{-0.15\ \text{J}}$$

This is neg. as expected.