Physics 221, Fall 2009

Quiz 3, Form: $\boxed{A}$

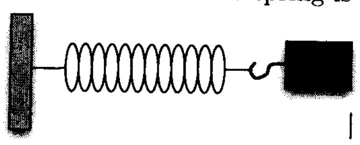
Name: Key

Date: ____________

Avogadro's Number: 6.02×10^{23} atoms/mol

Section 1. Multiple Choice

Questions 1–6: A horizontal spring and force sensor are set up as shown below. $x = 0$ is defined at the location shown when the spring is unstretched.



You pull the force sensor to the right until the magnitude of the force by the spring on the sensor is 10.0 N. Then, you hold it in in place with the spring stretched. You measure the distance stretched and find it to be 0.2 m.

1. What is the stiffness of the spring?
   (a) 2 N/m
   (b) 10 N/m
   (c) 20 N/m
   (d) 50 N/m
   (e) 500 N/m

$k = \frac{F}{s} = 50$

2. At this instant, what is the direction of the force by the spring on the sensor?
   (a) $< 0, +1, 0 >$
   (b) $< 0, -1, 0 >$
   (c) $< +1, 0, 0 >$
   (d) $< -1, 0, 0 >$

$\vec{F}_{by\ spring\ on\ sensor}$

spring is stretched so it "pulls"

3. What is the direction of the force by the sensor on the spring?
   (a) $< 0, +1, 0 >$
   (b) $< 0, -1, 0 >$
   (c) $< +1, 0, 0 >$
   (d) $< -1, 0, 0 >$

$\vec{F}_{by\ sensor\ on\ spring}$

Newton's 3rd law

4. What is the force by the bracket on the spring?
   (a) $< 0, +1, 0 >$
   (b) $< 0, -1, 0 >$
   (c) $< +1, 0, 0 >$
   (d) $< -1, 0, 0 >$ ← direction only!

10 N

$\vec{F}_{by\ bracket\ on\ spring}$

$\vec{F}_{by\ bracket} = \langle -10, 0, 0 \rangle N$

$\vec{F}_{net} = 0$ since it is in equilibrium

5. Suppose that you attach a second identical spring in series with the first one, and you again pull with the force sensor until it reads 10.0 N. You hold it in place. How far is the set of springs stretched (as measured from its unstretched length)?

(a) 0.05 m
(b) 0.8 m
(c) 0.4 m
(d) 0.2 m
(e) 0.1 m

$k_{eff} = \frac{k}{2}$

$F = k_{eff} s \qquad s = \frac{F}{k_{eff}} = \frac{F}{\frac{k}{2}} = 2\frac{F}{k}$

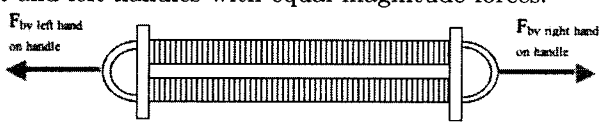
6. You remove the second spring and reattach it so that it is in parallel with the first spring. They are identical springs. You again pull the force sensor until it reads 10.0 N. You hold it in place. How far is each spring stretched (from its unstretched length)?

(a) 0.05 m
(b) 0.8 m
(c) 0.4 m
(d) 0.2 m
(e) 0.1 m

$k_{eff} = 2k$

$F = k_{eff} s = 2ks \qquad s = \frac{F}{2k} = \frac{1}{2}\frac{F}{k}$

Questions 7–8: A piece of exercise equipment consists of two parallel springs, as shown below. You pull the right and left handles with equal magnitude forces.



$F_1 = F_2 \qquad F = 50\,N$

Each spring has a stiffness of 100 N/m.

7. If you exert a force of magnitude 50 N on each handle, how far does each spring stretch?

(a) 0.5 m
(b) 0.25 m
(c) 0.4 m
(d) 0.1 m
(e) 0.125 m

$-F_1 + -F_2 = 50\,N$
$-2F_1 = 50\,N$
$F_1 = -25\,N$

$s = \frac{F}{k} = \frac{25\,N}{100\,\frac{N}{m}} = 0.25\,m$

8. Suppose that you apply a certain magnitude force F to each handle with your hands, and you cause the springs to stretch 10 cm. You then tie one handle to a wall, and pull the other handle with the same force F. In this case, the springs will

(a) stretch the same amount, 10 cm.
(b) stretch half as far, 5 cm.
(c) stretch twice as far, 20 cm.
(d) stretch less than 10 cm, but not exactly 5 cm.
(e) stretch more than 10 cm, but not exactly 20 cm.

$F \qquad F$

F

$F_{by\ bracket\ on\ spring}$

In both cases, $\vec{F}_{net} = 0$.

$F_{by\ bracket,x} = -10\,N$ just like $F_{by\ left\ hand}$

9. A child of mass 50 kg hangs by one arm from the monkey bars at the school playground. What is the tension in her arm?

   (a) 980 N
   (b) 245 N
   (c) 346 N
   (d) 693 N
   (e) 490 N

$F_{by\ arm}$ $\qquad$ $\vec{F}_{net} = 0$

$y: \quad 0 = F_{by\ arm\ y} - mg$

F_{grav} $\qquad$ $F_{by\ arm\ y} = mg = 490\ N$

10. If the child in the previous question hangs from two arms, with both arms vertical above her shoulders, what is the tension in each arm?

   (a) 980 N
   (b) 245 N
   (c) 346 N
   (d) 693 N
   (e) 490 N

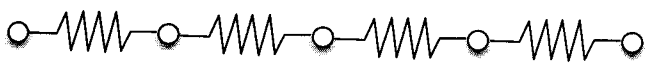
$F_{left\ arm}$ $\quad$ $F_{right\ arm}$ $\quad$ F_{grav}

$F_{left\ arm\ y} + F_{right\ arm\ y} = 490N$

$2F_{left\ arm\ y} = 490N$

$F_{left\ arm} = 245\ N$

An object of mass M is pulled by a rod of mass m. Model the rod as a 1-D chain of balls connected by springs. The rod's mass is NOT small compared to the object.



11. If the object speeds up while moving to the right, which atomic bonds in the rod are stretched the most?

   (a) The bonds on the right side of the rod.
   (b) The bonds on the left side of the rod.
   (c) Neither; the bonds are stretched the same throughout the rod.

$F_{T\ left} \longrightarrow F_{T\ right}$

$F_{T\ right} > F_{T\ left}$. Since $\vec{F}_{net} \longrightarrow$

12. If the object moves at a constant velocity, then the net force on the rod (and object) is zero. Which atomic bonds in the rod are stretched the most?

   (a) The bonds on the right side of the rod.
   (b) The bonds on the left side of the rod.
   (c) Neither; the bonds are stretched the same throughout the rod.

$F_{net} = 0$

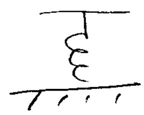
$F_{T\ left} \longleftarrow\longrightarrow F_{T\ right}$

$\vec{F}_{T\ right} = -\vec{F}_{T\ left}$

Questions 13–14: A platform that used to vault gymnasts in gymnastics can be modeled as a single vertical spring of stiffness 1×10^4 N/m. Suppose that when a gymnast lands on the vault, she lands on it vertically. Consider only her vertical motion. The gymnast's mass is 50 kg.

13. Suppose that the gymnast runs and jumps on the vault. At its lowest point, the spring in the platform is compressed 15 cm from its unstretched length. What is the force by the spring on the gymnast at this instant?

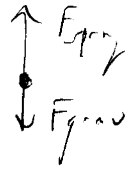
    (a) $< 0, 6.7 \times 10^4, 0 >$ N
    (b) $< 0, 1500, 0 >$ N
    (c) $< 0, 670, 0 >$ N
    (d) $< 0, 490, 0 >$ N
    (e) $< 0, 1010, 0 >$ N



$\vec{F}_{spring} = \langle 0, ks, 0 \rangle$
$= \langle 0, 1500, 0 \rangle$ N

14. For the previous question, what is the net force on the gymnast at this instant (when the spring is compressed 15 cm and is at its lowest point of the compression)?

    (a) $< 0, 6.7 \times 10^4, 0 >$ N
    (b) $< 0, 1500, 0 >$ N
    (c) $< 0, 670, 0 >$ N
    (d) $< 0, 490, 0 >$ N
    (e) $< 0, 1010, 0 >$ N



$\vec{F}_{spring} + \vec{F}_{grav} = \vec{F}_{net}$
$F_{net,y} = ks - mg$
$= 1500 - 490 = 1010$ N

15. One mole of silicon atoms has a mass of 28 grams, and its density is 2.4 grams per cubic centimeter. What is the approximate diameter of an silicon atom if you assume that the lattice structure of ~~iron~~ silicon is simple cubic?

    (a) 2.3×10^{-10} m
    (b) 2.5×10^{-10} m
    (c) 2.7×10^{-10} m
    (d) 2.8×10^{-10} m
    (e) 2.1×10^{-10} m

$V_{atom} = \left(\frac{1\ cm^3}{2.4\ g}\right)\left(\frac{28\ g}{mol}\right)\left(\frac{1\ mol}{6.02 \times 10^{23}\ atoms}\right)$
$= 1.94 \times 10^{-23}\ cm^3$
$d = V^{1/3} = 2.69 \times 10^{-8}$ cm
$\boxed{d = 2.69 \times 10^{-10}\ m}$

16. Tungsten wire is used in filaments of light bulbs. If Young's Modulus for tungsten is 3.6×10^{11} N/m^2 and tungsten's diameter is 2.51×10^{-10} m, what is the approximate bond stiffness for tungsten?

    (a) 90 N/m
    (b) 5 N/m
    (c) 14 N/m
    (d) 30 N/m
    (e) 70 N/m

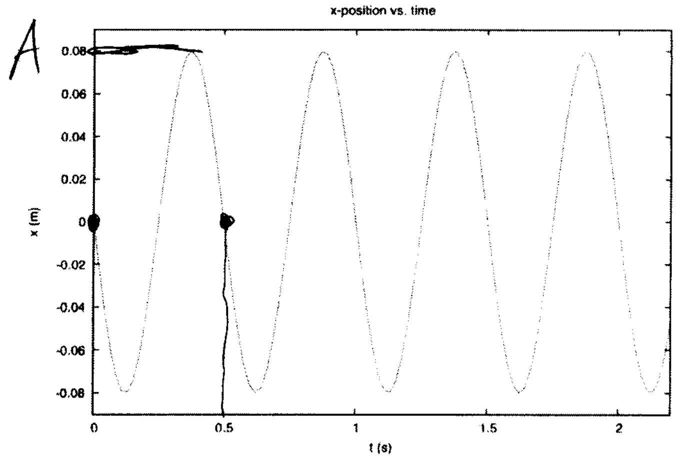
$k = Yd$
$= \left(3.6 \times 10^{11} \frac{N}{m^2}\right)\left(2.51 \times 10^{-10}\ m\right)$
$= 90 \frac{N}{m}$

17. The diameter of a copper atom is about 2.3×10^{-10} m. How many copper atoms are in 1 cm^3 of copper?

    (a) 8.2×10^{28} atoms
    (b) 8.2×10^{22} atoms
    (c) 1.2×10^{28} atoms
    (d) 1.2×10^{22} atoms
    (e) 1.9×10^{22} atoms

$d = 2.3 \times 10^{-8}$ cm
$V_{atom} = d^3 = 1.22 \times 10^{-23}\ cm^3$
$\#\ atoms = \frac{V_{total}}{V_{atom}} = \frac{1\ cm^3}{1.22 \times 10^{-23}\ cm^3} = 8.2 \times 10^{22}$ atoms

Questions 18–19: A graph of x vs. t for an oscillating mass on a spring is shown below.



18. What is the amplitude?

    (a) 0.5 m
    (b) 0.04 m
    (c) 0.16 m
    (d) 0.08 m
    (e) 0.25 m

peak

19. What is the period?

    (a) 0.08 s
    (b) 0.25 s
    (c) 1.0 s
    (d) 0.5 s
    (e) 0.04 s

Δt for 1 cycle

$T = 0.5\ s$

A 0.4-kg object hangs in equilibrium from a spring of stiffness 20 N/m. You pull it down 2 cm from equilibrium and release it from rest.

20. What is the angular frequency?

    (a) 50 rad/s
    (b) 7.1 rad/s
    (c) 0.14 rad/s
    (d) 0.2 rad/s
    (e) 2.8 rad/s

$\omega = \sqrt{\frac{k}{m}} = 7.1\ \frac{rad}{s}$

21. What is the period?

    (a) 0.022 s
    (b) 0.090 s
    (c) 0.22 s
    (d) 2.5 s
    (e) 0.88 s

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi} = \frac{7.1}{2\pi} = 1.12\ Hz$

$T = \frac{1}{f} = 0.89\ s$

22. If you double the mass of the oscillator, by what factor does the angular frequency change?

(a) $1/\sqrt(2)$
(b) $1/2$
(c) $\sqrt{2}$
(d) 2
(e) 1; the angular frequency stays the same; it does not depend on mass.

$\omega = \sqrt{\frac{k}{m}}$ $\omega \propto \frac{1}{\sqrt{m}}$

$\frac{1}{\sqrt{2m}} = \left(\frac{1}{\sqrt{2}}\right)\frac{1}{\sqrt{m}}$

23. If you double the stiffness of the oscillator, by what factor does the angular frequency change?

(a) $1/\sqrt(2)$
(b) $1/2$
(c) $\sqrt{2}$
(d) 2
(e) 1; the angular frequency stays the same; it does not depend on stiffness.

$\omega \propto \sqrt{k}$

$\sqrt{2k} = (\sqrt{2})\sqrt{k}$

24. If you double the amplitude of the oscillator, by what factor does the angular frequency change?

(a) $1/\sqrt(2)$
(b) $1/2$
(c) $\sqrt{2}$
(d) 2
(e) 1; the angular frequency stays the same; it does not depend on amplitude.

ω is independent of A

25. If you double the amplitude, by what factor does the maximum speed of the oscillator change as it passes through equilibrium?

(a) 2
(b) $\sqrt{2}$
(c) $1/2$
(d) 4
(e) 1; the maximum speed of the oscillator stays the same; it does not depend on amplitude.

$V_{max} = \omega A$

$2A$ gives $2 V_{max}$

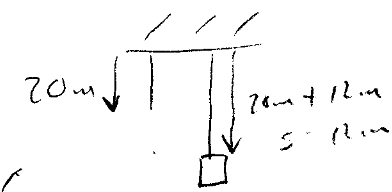
+25

Section 2. Problem Solving

Select two pages out of the following three pages to solve. Only two will be graded. Circle the question numbers of the ones that you want me to grade.

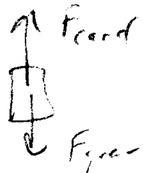
26. A slack bungee cord has a length of 20 m. When a 80-kg bungee jumper hangs from the cord and remains at rest in equilibrium, the cord stretches 12 m from its unstretched length.

Sketch a picture showing a slack cord and the stretched cord side-by-side with mass hanging from the stretched cord. Indicate distances in your picture. You must be able to visualize the situation in order to answer the questions below.



(a) What is the stiffness of the bungee cord? (You must show all of your work, starting with the Momentum Principle (Newton's 2nd law). If you do not use logical steps starting from Newton's second law, then you will not get full credit for your answer.)

+5



$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t} = 0$

$F_{cord_y} - |\vec{F}_{grav}| = 0$

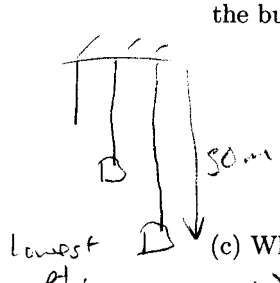
$F_{cord_y} = |\vec{F}_{grav}|$

$ks = mg$

$k = \frac{mg}{s} = \frac{(80)(9.8)}{12} = 65 \frac{N}{m}$

(b) Suppose that using this same bungee cord, a bungee jumper jumps from a bridge. She falls for 20 m before the cord starts to stretch. At her lowest point during her motion, she comes to a stop and rebounds upward. If at her lowest point, she is 50 m below the bridge where she jumped. How far is the bungee cord stretched (from its unstretched length) at this instant?

+5

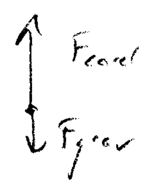


s at lowest pt. is

$s = 50 - 20 = 30\,m$

(c) What is the net force on her at her lowest point during the jump?

+5



$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t} = \vec{F}_{cord} + \vec{F}_{grav}$

$F_{net_y} = ks - mg = (65)(30) - 80(9.8) = 1166\,N$

(d) What is her momentum 0.1 s later as she is rebounding upward?

+5

$\vec{p}_f = \cancel{\vec{p}_i} + \vec{F}_{net}\Delta t$

0 at lowest pt.

$\vec{p}_f = \vec{F}_{net}\Delta t = (\langle 0, 1166, 0\rangle N)(0.1 s) = \langle 0, 117, 0\rangle\, kg\,\frac{m}{s}$

(e) What is her position 0.1 s later as she is rebounding upward?

+5

$\vec{v}_f = \frac{\vec{p}_f}{m} = \langle 0, 1.46, 0\rangle \frac{m}{s}$

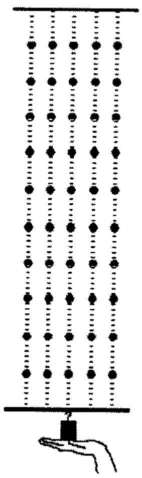
use $\vec{v}_{avg} \approx \vec{v}_f$ for small Δt

$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t = \langle 0, -50, 0\rangle + \langle 0, 1.46, 0\rangle(0.1) = -49.9\,m$ below bridge.

She only rose 0.1 m in 0.1 s.

+25

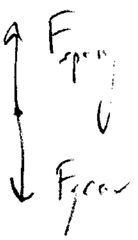
27. In lab, you build an apparatus with balls connected by springs like the one shown below. The mass of the balls and springs are negligible.



If you hang a 0.5-kg object from the bottom of the apparatus in equilibrium, it stretches 1 cm.

(a) If you model the apparatus as a single spring, what is the stiffness of the apparatus?

+5



$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t} = 0$

$\vec{F}_{net} = \vec{F}_{spring} + \vec{F}_{grav}$

y: $0 = ks - mg$

$k = \frac{mg}{s} = \frac{(0.5)(9.8)}{0.01} = 490\ \frac{N}{m}$

(b) What is the stiffness of a single chain (or strand) of balls and springs?

+10

$k_{eff} = 490\frac{N}{m}$ 5 strands.

In parallel: $k_{eff} = Nk$

$k_{strand} = \frac{k_{eff}}{5} = \frac{490}{5} = 98\frac{N}{m}$

+10

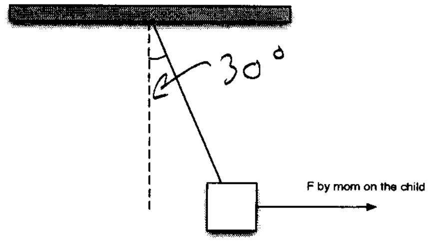
(c) What is the stiffness of a single spring?

In series: $k_{eff} = \frac{k}{N}$ where k_{eff} is $k_{strand} = 98\ \frac{N}{m}$

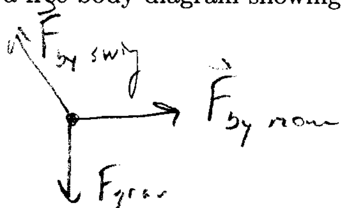
11 springs

$k_{spring} = k_{eff} N = (98\ \frac{N}{m})(11) = \boxed{1080\frac{N}{m}}$

28. A mom pushes her child who is sitting on a swing in equilibrium as shown below. Mom pushes the swing to the right horizontally with a constant force. The swing makes an angle of 30° with respect to the vertical and it is in equilibrium. The mass of the child and seat is 25 kg. Define the system to be the child and the seat.



(a) Sketch a free-body diagram showing all forces on the system.



+5

(b) What is the force by Mom on the system and what is the tension in the swing (chain)?

+20

$$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t} = 0$$

$$\vec{F}_{net} = \vec{F}_{swing} + \vec{F}_{mom} + \vec{F}_{grav}$$

x: $F_{swing\,x} + F_{mom\,x} = 0$

$F_{swing\,x} = -F_{mom\,x}$

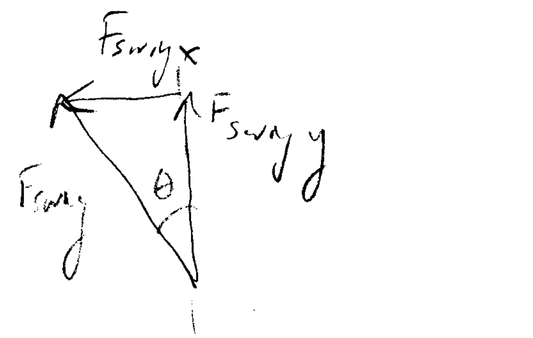
y: $F_{swing\,y} - mg = 0$

$F_{swing\,y} = mg$

$F_{swing}\cos(30) = mg$

$F_{swing} = \frac{mg}{\cos 30} = \frac{(25)(9.8)}{\cos 30}$

$\boxed{F_{swing} = 283\ N}$ tension!



$F_{swing\,x} = F_{swing}\cos(90+30)$

$= F_{swing}\cos(120)$

$F_{swing\,y} = F_{swing}\cos(30)$

$F_{swing\,x} = (283\ N)\cos(120)$

$= -142\ N$

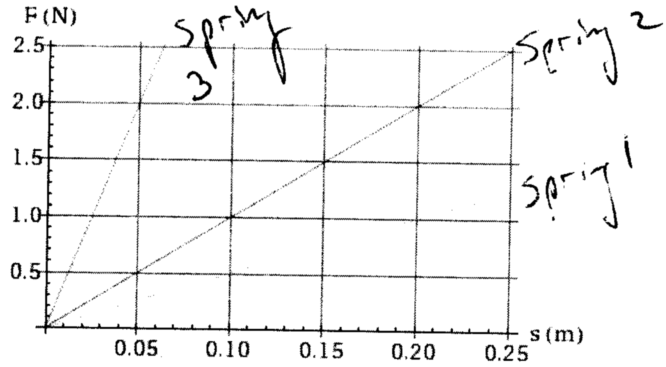
$F_{mom\,x} = -F_{swing\,x} = 142\ N$

$\boxed{F_{mom} = 142\ N}$

+10

Section 3. Lab

29. Three springs of different stiffness are shown below. Which spring is easiest to stretch?



$\boxed{\text{Spring 1}}$

30. What is the stiffness of spring 2 in the previous question?

$$\text{slope} = \frac{F}{s} = \frac{0.5\,N}{0.05\,m} = 10\,\frac{N}{m}$$

31. A spring of stiffness 10 N/m and a spring of stiffness 30 N/m are attached in series. What is the effective stiffness of the system of springs?

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k_{eff} = \left(\frac{1}{10} + \frac{1}{30}\right)^{-1} = \frac{30}{4} = \boxed{7.5\,\frac{N}{m}}$$

less stiff!

32. A spring of stiffness 10 N/m and a spring of stiffness 30 N/m are attached in parallel. What is the effective stiffness of the system of springs?

$$k_{eff} = k_1 + k_2 = 10 + 30 = \boxed{40\,\frac{N}{m}}$$

Suppose that in lab, you study an oscillating mass-spring system, and you measure the position of the object as a function of time. A curve fit shows that:
$x(t) = 0.025\cos(8.0t + 1.1)$

33. What is the amplitude? $\boxed{A = 0.025\,m}$

34. What is the angular frequency? $\boxed{\omega = 8.0\,\frac{rad}{s}}$

35. What is the period?

$$T = \frac{2\pi}{\omega} = \boxed{0.79\,s}$$

36. Write an equation for the x-velocity as a function of time, $v_x(t)$.

$$V_x = \frac{dx}{dt} = -8(0.025)\sin(8t+1.1) = \boxed{-0.2\sin(8t+1.1)}$$

37. What is the speed of the object as it passes through equilibrium? $\boxed{V_{max} = 0.2\,\frac{m}{s}}$

38. Write an equation for $F_{net,x}$ as a function of time, $F_{net,x}(t)$.

$\vec{p} = m\vec{v}$ $\quad p_x = m(-0.2)\sin(8t+1.1)$

39. What is the maximum net force on the object and where is the object when the net force on the object is a maximum?

$$F_{net_x} = \frac{dp_x}{dt} = -m(0.2)(8)\cos(8t+1.1)$$
$$= -m(1.6)\cos(8t+1.1)$$

$\boxed{F_{net_{max}} = 1.6\,m}$

It is a max at $x = -A$
The object is at its min. x. (spring is most compressed!)