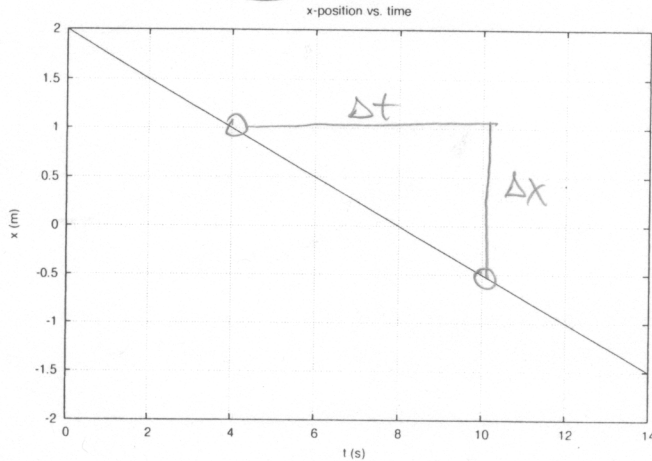


Use  $g = 9.8 \text{ N/kg}$  for the magnitude of Earth's gravitational field near Earth's surface. Define the  $+x$  axis to the right and the  $+y$  axis toward the top of the page. All answers (except multiple choice) must be accompanied by mathematics or an explanation of reasoning. An answer with no reasoning is unacceptable. Numeric answers must include units. Sketches must be labeled.

Section 1. Multiple Choice

Questions 1-4: The  $x$  vs.  $t$  graph for a steel ball rolling on a level track is shown below.



slope is  $v_x$  on  $x$  vs.  $t$  graph

$v_x$  is constant

$$\text{slope} = \frac{-0.5 - 1}{10 - 4}$$

$$= \frac{-1.5 \text{ m}}{6 \text{ s}}$$

$$= -0.25 \frac{\text{m}}{\text{s}}$$

- The best description of the ball's motion is
  - The ball rolls with uniform motion to the right.
  - The ball rolls with uniform motion to the left.
  - The ball is speeding up while moving to the left.
  - The ball is slowing down while moving to the left.
  - The ball is rolling down a hill.

- The net force on the ball is
  - increasing at a constant rate, in the  $-x$  direction.
  - decreasing at a constant rate, in the  $-x$  direction.
  - constant, and in the  $-x$  direction.
  - constant, and in the  $+x$  direction.
  - zero.

$v_x$  is constant, so

$$F_{\text{net } x} = 0.$$

Newton's First Law!

- What is the  $x$ -velocity of the ball?
  - zero
  - 2.0 m/s
  - 1.5 m/s
  - 0.25 m/s
  - 8.0 m/s

$$\text{slope} = -0.25 \frac{\text{m}}{\text{s}}$$

4. What is the x-displacement of the ball between  $t = 4$  s and  $t = 12$  s?

- (a) 1 m
- (b) -2 m
- (c) -1 m
- (d) 8 m
- (e) -0.5 m

read values  
of x  
from graph

$$x_f = -1, \quad x_i = 1$$
$$\Delta x = -1 - 1 = -2 \text{ m}$$

Questions 5-7: The net force on an object is

$$\vec{F}_{net} = \langle -4.0, 6.0, 7.0 \rangle \text{ N.}$$

5. What is the magnitude of the net force on the object?

- (a)  $|\vec{F}_{net}| = 5.00$  N
- (b)  $|\vec{F}_{net}| = 14.2$  N
- (c)  $|\vec{F}_{net}| = 7.21$  N
- (d)  $|\vec{F}_{net}| = 5.39$  N
- (e)  $|\vec{F}_{net}| = 10.0$  N

$$|\vec{F}_{net}| = 10 \text{ N}$$
$$= \sqrt{4^2 + 6^2 + 7^2}$$
$$= 10 \text{ N}$$

6. What is the unit vector for the net force in the previous question?

- (a)  $\langle -1, 1, 1 \rangle$
- (b)  $\langle -0.20, 0.30, 0.35 \rangle$
- (c)  $\langle -0.44, 0.67, 0.78 \rangle$
- (d)  $\langle -0.40, 0.60, 0.70 \rangle$
- (e)  $\langle -1, 0, 0 \rangle$

$$\hat{F}_{net} = \frac{\vec{F}_{net}}{|\vec{F}_{net}|} = \frac{\langle -4, 6, 7 \rangle \text{ N}}{10 \text{ N}}$$
$$= \langle -0.4, 0.6, 0.7 \rangle \text{ N}$$

7. What angle does the force vector make with the +x axis?

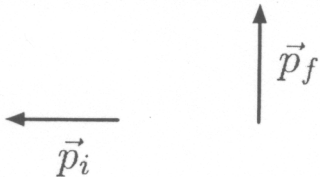
- (a)  $34^\circ$
- (b)  $114^\circ$
- (c)  $204^\circ$
- (d)  $53^\circ$
- (e)  $0^\circ$

"direction cosine"

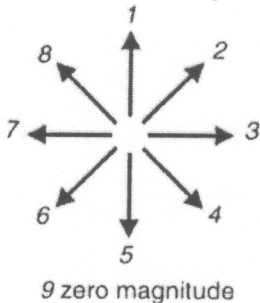
$$\cos \theta_x = \frac{F_{net,x}}{|\vec{F}_{net}|} = \frac{-4}{10} = -0.4$$

$$\theta_x = 114^\circ$$

8. A baseball has an initial momentum in the  $-x$  direction when it collides with a baseball bat and "pops" straight upward in the  $+y$  direction. The initial and final momentum vectors for the baseball before and after the collision are shown below.



Which arrow below points in the direction of the change in momentum,  $\Delta\vec{p}$ , of the baseball?



Draw  $\vec{p}_i$  and  $\vec{p}_f$  tail to tail

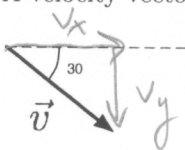
- (a) 2
- (b) 8
- (c) 3
- (d) 1
- (e) 6

9. A projectile is launched at an angle of  $\theta$  above the horizontal. When the projectile is at its peak,

- (a)  $|\vec{v}| = v_{ix}$  and  $|\vec{F}_{net}| = mg$
- (b)  $|\vec{v}| = v_{ix}$  and  $|\vec{F}_{net}| = 0$
- (c)  $|\vec{v}| = 0$  and  $|\vec{F}_{net}| = mg$
- (d)  $|\vec{v}| = 0$  and  $|\vec{F}_{net}| = 0$



10. A velocity vector for a projectile is shown below. The projectile's speed at this instant is 10.0 m/s.



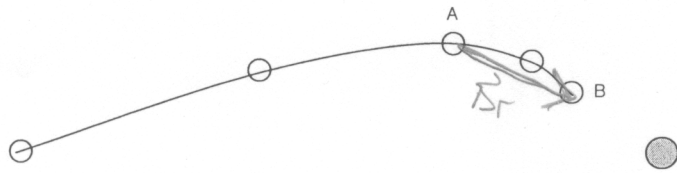
$$v_x = |\vec{v}| \cos(-30) = 10 (0.866) = 8.66 \frac{m}{s}$$

$$v_y = |\vec{v}| \cos(-120) = 10 (-0.5) = -5 \frac{m}{s}$$

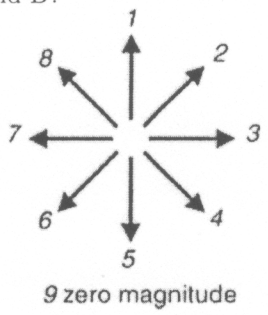
Write the vector in component form.

- (a)  $\vec{v} = \langle +8.66, -5.00, 0 \rangle$  m/s
- (b)  $\vec{v} = \langle +5.00, +8.66, 0 \rangle$  m/s
- (c)  $\vec{v} = \langle +8.66, +5.00, 0 \rangle$  m/s
- (d)  $\vec{v} = \langle +5.00, -8.66, 0 \rangle$  m/s
- (e)  $\vec{v} = \langle +10.0, -10.0, 0 \rangle$  m/s

Questions 11-12: A golf ball rolls on the green toward the hole. Its motion is affected by the slope of the green as well as friction. Its path is shown below, as it moves from left to right.



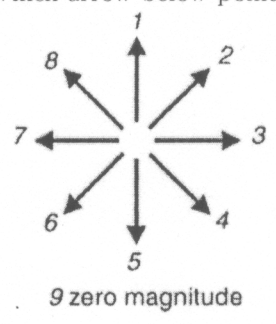
11. Which arrow below points in the direction of the *average* velocity of the golf ball between points A and B?



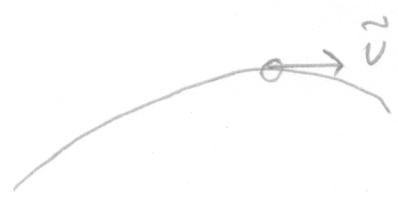
$\vec{\Delta r}$  and  $\vec{v}_{avg}$  are in same dir.  
 since  $\vec{v}_{avg} = \frac{\vec{\Delta r}}{\Delta t}$

- (a) 3
- (b) 2
- (c) 5
- (d) 4
- (e) 1

12. Which arrow below points in the direction of the *instantaneous* velocity of the golf ball at point A?

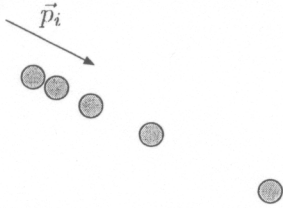


$\vec{v}$  is tangent to the path

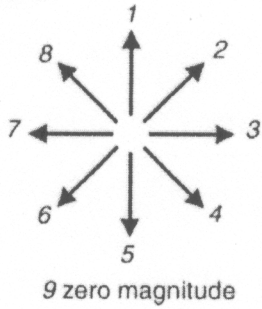


- (a) 3
- (b) 2
- (c) 5
- (d) 4
- (e) 1

13. You use video analysis to study the motion of an object, and you mark the position of the object at equal time intervals, as shown below. The initial momentum of the object is in the direction shown.



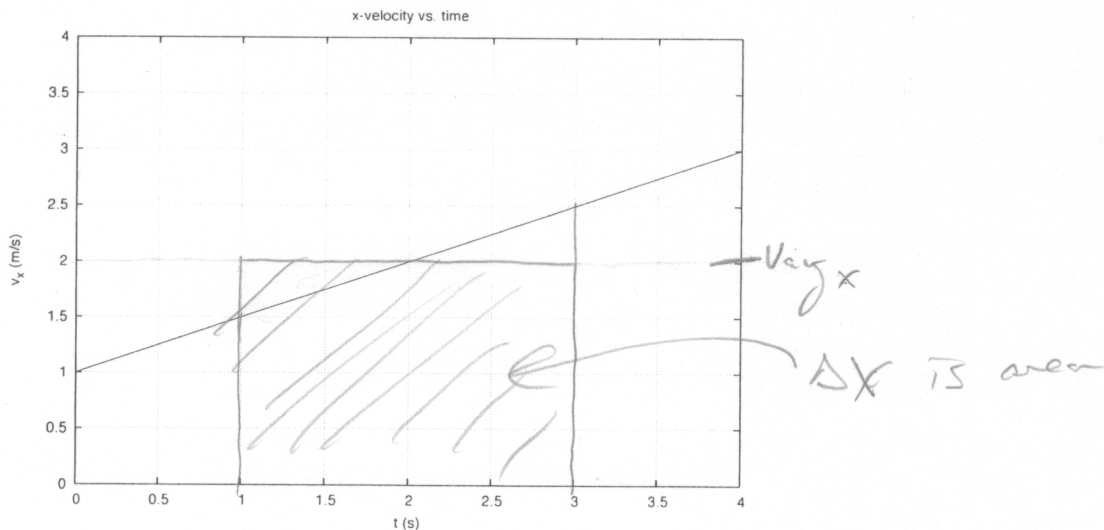
Which arrow below points in the direction of the net force on the object during the time interval shown?



speeding up so  $\vec{F}_{net}$  and  $\vec{p}$  are in the same direction

- (a) 8
- (b) 3
- (c) 4
- (d) 5
- (e) 9

Questions 14–17: An object's x-velocity vs. time graph is shown below.



14. What is the average x-velocity of the object during the time interval from  $t = 1$  s to  $t = 3$  s?

- (a) 0.5 m/s
- (b) 1.0 m/s
- (c) 1.5 m/s
- (d) 2.0 m/s
- (e) 2.5 m/s

$$v_{ix} = 1.5 \frac{m}{s} \quad v_{fx} = 2.5 \frac{m}{s}$$

$$v_{avgx} = \frac{2.5 + 1.5}{2} = 2 \frac{m}{s}$$

15. What is the x-displacement of the object during the time interval from  $t = 1$  s to  $t = 3$  s?

- (a) 3.0 m
- (b) 2.0 m
- (c) 5.0 m
- (d) 1.0 m
- (e) 4.0 m

$$\begin{aligned}\Delta x &= \text{area under curve} \\ &= v_{\text{avg},x} \Delta t \\ &= \left(\frac{2}{3} \text{ m/s}\right)(2 \text{ s}) = 4 \text{ m}\end{aligned}$$

16. Suppose that when the clock reads  $t = 1$  s, the object is at the location  $x = 10$  m. What is its position at  $t = 3$  s?

- (a) 15 m
- (b) 14 m
- (c) 12 m
- (d) 20 m
- (e) 2.5 m

$$\begin{aligned}x_f &= x_i + v_{\text{avg},x} \Delta t \quad \text{"position update"} \\ &= 10 \text{ m} + \left(\frac{2}{3}\right)(2 \text{ s}) = 10 \text{ m} + 4 \text{ m} \\ &= 14 \text{ m}\end{aligned}$$

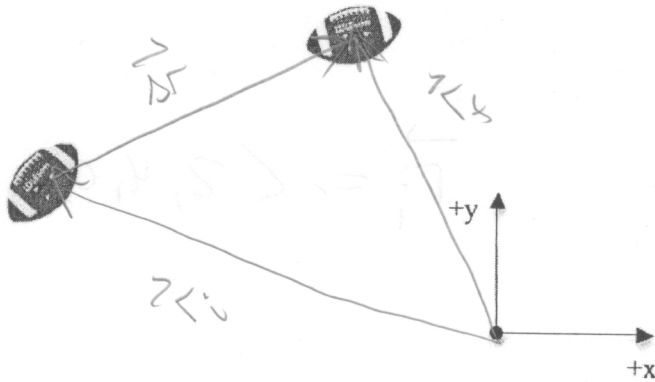
17. The x-velocity vs. time graph shows us that the net force on the object is

- (a) increasing, in the  $-x$  direction
- (b) constant, in the  $-x$  direction
- (c) constant, in the  $+x$  direction
- (d) increasing, in the  $+x$  direction
- (e) zero

because slope is constant  
and is +.

Section 2. Sketching Vectors

18. A football travels from left to right in the picture shown below.



Using the given origin, sketch the following vectors, with labels, on the figure above:

- initial position,  $\vec{r}_i$
- final position,  $\vec{r}_f$
- displacement,  $\Delta\vec{r}$

19. A spaceship in a simulation has a velocity  $\langle -10, -10, 0 \rangle$  m/s at  $t=0$  when a thruster fires and exerts a constant net force of  $\langle 0, 100, 0 \rangle$  N on the spaceship. The spaceship at  $t=0$  is shown below. Sketch the following items on the figure. Label the vectors.

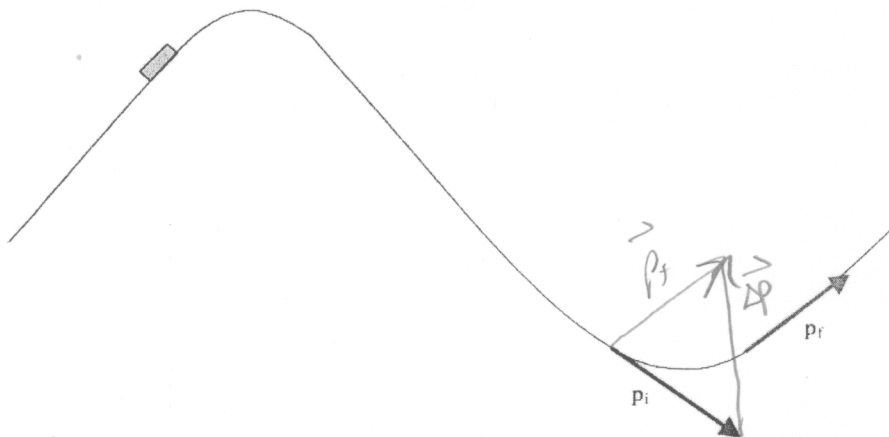
- the initial velocity vector,  $\vec{v}_i$
- the net force on the spaceship,  $\vec{F}_{net}$
- the path of the spaceship,  $\Delta\vec{r}$



- $v_x$  is constant since  $F_{net,x} = 0$
  - parabola (since  $\vec{F}_{net}$  is constant)
  - parabola must be tangent to  $\vec{v}_i$  at beginning and curve toward  $\vec{F}_{net}$
- (i.e. a top-opening parabola in this case)

Note: rotate page  $180^\circ$  and it looks like a projectile!

20. A roller coaster goes up a hill and then down a hill as shown in the figure below. Its momentum just before the bottom of the hill and just after the bottom of the hill are shown. Sketch the change in momentum of the roller coaster during this time interval.



### Section 3. Analytic Problems

21. A tennis ball of mass 0.057 kg travels with velocity  $\langle 50, 0, 0 \rangle$  m/s toward a wall. After bouncing off the wall, the tennis ball is observed to be traveling with velocity  $\langle -45, 0, 0 \rangle$  m/s. The tennis ball was in contact with the wall for 0.10 s. What was the net force on the tennis ball due to the collision with the wall? (Note: the net force is due to the force by the wall on the ball. The gravitational force on the ball by Earth is negligible in this case.)

$$\vec{F}_{\text{net}} = \frac{\Delta \vec{p}}{\Delta t} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$$

$$= \frac{(0.057 \text{ kg})(\langle -45, 0, 0 \rangle - \langle 50, 0, 0 \rangle) \text{ m/s}}{0.1 \text{ s}}$$

$$= \langle -54, 0, 0 \rangle \text{ N}$$

22. Assuming that the force by the wall on the ball is constant, then the time interval that the ball is slowing down and being compressed (during the collision) is  $1/2(0.1 \text{ s}) = 0.05 \text{ s}$ . How much does the tennis ball compress during this time interval?

$$\vec{r}_f = \vec{r}_i + \vec{v}_{\text{avg}} \Delta t$$

$$x_f = x_i + v_{\text{avg},x} \Delta t$$

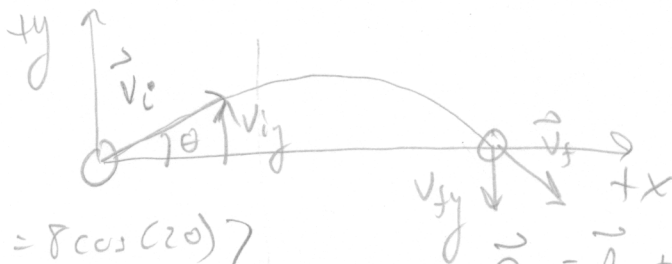
$$x_f = x_i + \left( \frac{v_{i,x} + v_{f,x}}{2} \right) \Delta t$$

$$x_f - x_i = \left( \frac{v_{i,x} + v_{f,x}}{2} \right) \Delta t$$

$$8\Delta x = \left( \frac{50 + 0}{2} \right) (0.05 \text{ s}) = \frac{1.25 \text{ m}}{\text{impossible}}$$



23. Two softball players are playing catch. One player throws the softball with a speed of 8 m/s at an angle of  $20^\circ$  above the horizontal. The other player catches it at the same height that the first player threw the ball. How far apart are the players, horizontally?



Note: due to symmetry,  
 $v_{fy} = -v_{iy}$

$$\left. \begin{aligned} v_{ix} &= 8 \cos(20) \\ &= 7.52 \frac{\text{m}}{\text{s}} \\ v_{iy} &= 8 \sin(20) \\ &= 2.73 \frac{\text{m}}{\text{s}} \end{aligned} \right\} \text{use direction cosines!}$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_{iy} \Delta t$$

$$x_f = x_i + v_{ix} \Delta t$$

$$\boxed{x_f = v_x \Delta t}$$

$$\boxed{\text{need } \Delta t!}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net} \Delta t$$

$$v_{fy} = v_{iy} + \frac{F_{net,y}}{m} \Delta t$$

$$v_{fy} = v_{iy} + \frac{-mg}{m} \Delta t$$

$$v_{fy} = v_{iy} - g \Delta t$$

$$-v_{iy} = v_{iy} - g \Delta t$$

$$2v_{iy} = g \Delta t$$

$$\Delta t = \frac{2v_{iy}}{g} = \frac{2(2.73)}{9.8} = \boxed{0.5575}$$

Thus,

$$x_f = v_x \Delta t$$

$$= (7.52 \frac{\text{m}}{\text{s}})(0.5575)$$

$$\boxed{x_f = 4.19 \text{ m}}$$

24. How high (above the player's hand) was the baseball when it was at its peak?



At peak,  $v_{fy} = 0$ .

$$\text{Also, } \Delta t = \frac{1}{2}(0.5575)$$

$$= 0.279 \text{ s}$$

Use  $\vec{r}_f = \vec{r}_i + \vec{v}_{iy} \Delta t$

$$y_f = y_i + \frac{v_{iy} + v_{fy}}{2} \Delta t$$

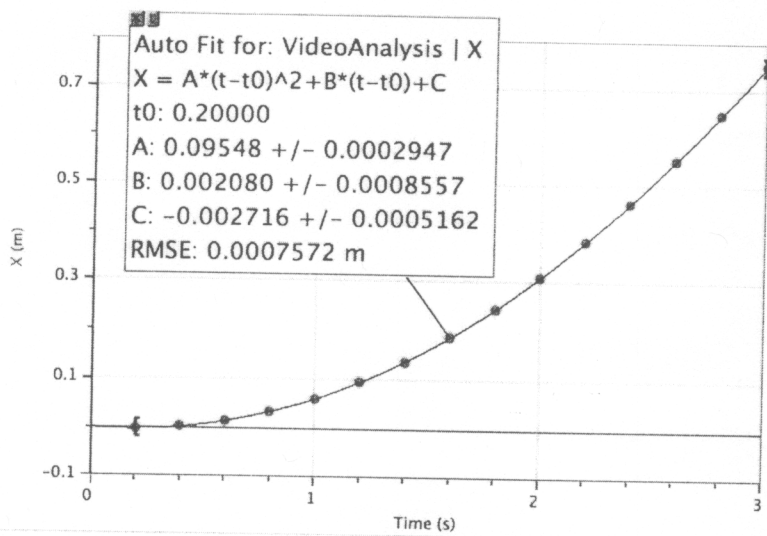
$$y_f = \frac{v_{iy} + v_{fy}}{2} \Delta t$$

$$= \left( \frac{2.73 + 0}{2} \right) (0.279 \text{ s})$$

$$\boxed{y_f = 0.381 \text{ m}}$$

Section 4. Lab Questions

25. The graph and the resulting curve fit for the x-position of a fancart as a function of time are shown below.



$$v_x = \frac{dx}{dt}$$

$$= 2A(t-t_0) + B$$

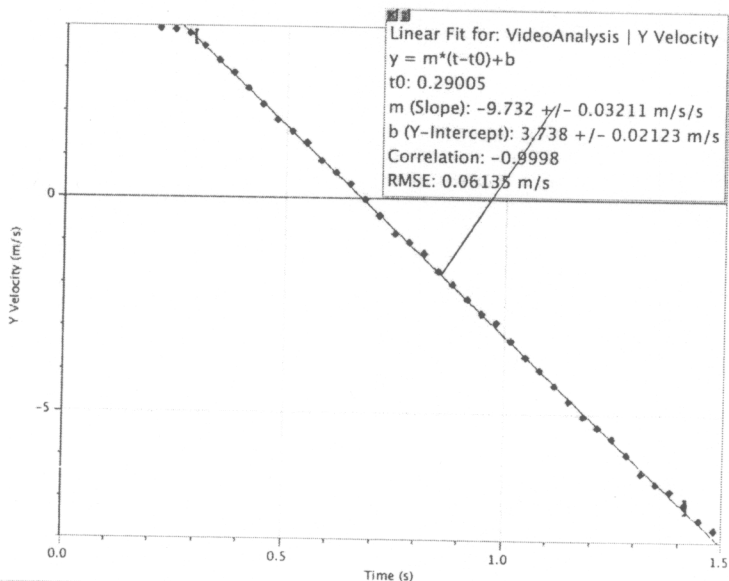
$$a_x$$

From the curve fit, determine the x-acceleration of the fancart. Be sure to show your work or explain your reasoning.

$$a_x = 2A$$

$$= 2(0.0955 \frac{m}{s^2}) = 0.191 \frac{m}{s^2}$$

26. In the video analysis of a basketball, you fit a curve to the y-velocity vs. time graph as shown below.



According to the best-fit function, what is the y-acceleration of the basketball?

$$a_y \equiv \text{slope} = -9.7 \frac{m}{s^2}$$

$\sigma_A$

$v_y$  vs.  $t$  graph

The VPython program below is a computer model for the motion of a punted soccer ball. Units for defined quantities are N, m, kg, s, and combinations thereof.

```
1 from __future__ import division
2 from visual import *
3 from visual.graph import *
4
5
6 grass = box(pos=vector(0, -0.05, 0), size=(20.0, 0.05, 10), color=color.green)
7 ball = sphere(pos=vector(-25, 0.5, 0), radius=0.4, color=color.white)
8
9 ball.m = 0.6
10 ball.v = vector(8, 6, 0)
11 ball.p = ball.m * ball.v
12 g = 9.8
13
14 dt = 0.01
15 t = 0
16
17 while ball.pos.y > 0:
18     rate(100)
19     Fnet=vector(0, -ball.m*g, 0)
20     ball.p = ball.p + Fnet*dt
21     ball.v = ball.p/ball.m
22     ball.pos = ball.pos + ball.v*dt
23     t = t+dt
```

1. What is the initial position of the ball?

See Line 7

$$\vec{r}_i = \langle -25, 0.5, 0 \rangle \text{ m}$$

2. What is the initial velocity of the ball?

See Line 10

$$\vec{v}_i = \langle 8, 6, 0 \rangle \frac{\text{m}}{\text{s}}$$

3. Which line number defines the net force on the ball?

See Line 19

19

4. Which line number updates the momentum of the ball?

20

5. Which line number updates the position of the ball?

21

6. In the position update equation, does it use the average velocity, the initial velocity, or the final velocity of the ball during the time interval  $dt$  to update the position of the ball?

It uses the final velocity because the momentum (and thus velocity) gets updated before the position. Thus, the position update is using the final velocity of the ball.