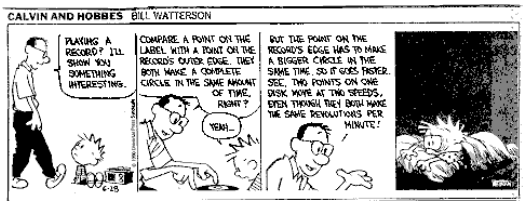
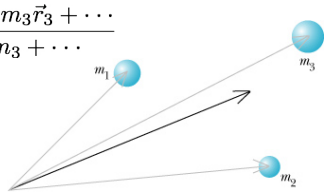
Chapter 8



Center of Mass

Treat the system as a single particle of mass M located at the center of mass of the system.

$$\vec{r}_{c.m.} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$



Newton's second law for a multiparticle system

Regardless of the motion of the individual particles, the center of mass motion is determined by the net external force on the system. Where forces are applied is inconsequential; you can treat all forces as if they act at the center of mass.

$$\vec{F}_{net} = \frac{d\vec{P}_{tot}}{dt}$$

Example: Binary Stars

Example -- Fireworks

A fireworks shell explodes into two smaller shells before reaching its peak. After the explosion, the peak reached by the center of mass of the system will be

1. Higher than the peak of the original shell if it had not exploded.
2. Lower than the peak of the original shell if it had not exploded.
3. Either higher or lower than the peak of the original shell if it had not exploded, depending on the conditions of the explosion.

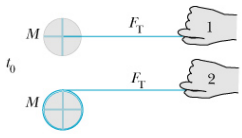
Poll

Diver A jumps from a 10-m platform and dives straight into the water. Diver B jumps with the same initial velocity as A but rotates three times before hitting the water. Which diver will hit the water first?

1. Diver A
2. Diver B
3. They will both hit the water at the same time.

Poll

For puck A, a string is attached to the center of the puck. For puck B, a string is wrapped around the puck and the string unwinds when pulled. The two strings are pulled by equal forces. Which puck will have a greater acceleration?



1. Puck A
2. Puck B
3. They will both have the same acceleration.

See VPython simulation

Kinetic Energy for a multiparticle system

The total kinetic energy is the sum of the kinetic energy of each particle in the system.

$$K_{tot} = \sum_{i=1}^{N} \frac{p_i^2}{2m} \quad \text{or} \quad K_{tot} = \sum_{i=1}^{N} \frac{1}{2} m v_i^2$$

which can be conveniently rewritten as

$$K_{tot} = \frac{1}{2} M v_{c.m.}^2 + \sum_{i=1}^{N} \frac{1}{2} m_i v_{i_{rel}}^2$$

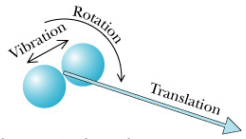
Breaking up the motion

Break up the motion into two parts:

(1) the motion of the center of mass (**translational** motion)

(2) the motion of the object relative to the center of mass (**relative** motion)

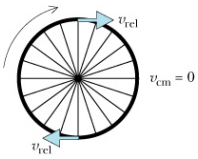
$$K_{tot} = K_{trans} + K_{rel}$$

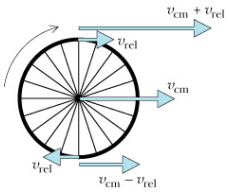


Relative motion can be vibrational or rotational or a combination of vibration and rotation, for example.

See diatomic molecule simulation. See video of toy.

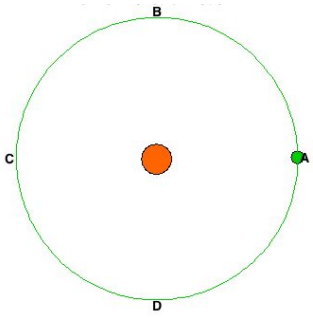
Example -- Spinning, Translating Wheel



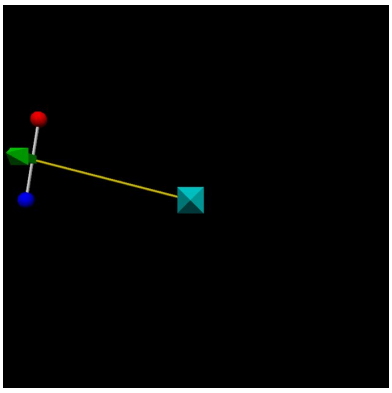


See wheel simulations

Example--A Planet



Example--A rotating barbell



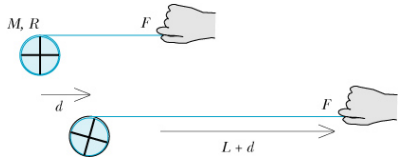
Conservation of Energy

Treat the system as a point particle to calculate translational kinetic energy.

Apply conservation of energy to the real system to calculate relative kinetic energy.

Example

A string is wrapped around a disk of mass M and radius R. Starting from rest, you pull the string with a constant force F. At the instant when the center of the disk has moved a distance d, a length L of string has unwound off the disk. What is the (center-of-mass) speed of the disk? What is its rotational kinetic energy?



Example

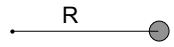
A yo-yo of radius R falls distance h from rest while your hand moves upward a distance d. What is its translational kinetic energy, its rotational kinetic energy, and its total kinetic energy at this instant?

Rotational kinetic energy

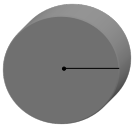
Rotational kinetic energy relative to the center of mass for a solid body depends on the angular speed of the system and how the mass is distributed about the center of mass.

For a point particle

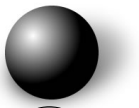
Moment of inertia (about center of mass)



Point particle $I = mR^2$



Solid wheel rotating about its center. $I = \frac{1}{2}MR^2$



Sphere rotating about its center. $I = \frac{2}{5}MR^2$

Thin hollow wheel rotating about its center $I = MR^2$

Example--Moment of Inertia

A 50-kg child is on a rotating merry-go-round of mass 300-kg and radius 2 m. If the child is at the edge of the merry-go-round, what is the total moment of inertia of the system?

Example--Kinetic Energy of Earth

What is the total kinetic energy of Earth?