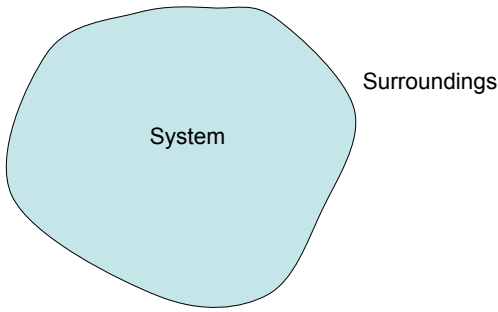
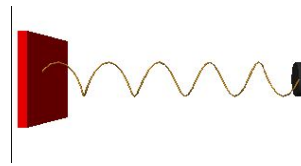




## Chapter 6--Potential Energy of a Spring



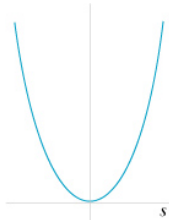
## Potential Energy of an Ideal Spring



$$F_x = -\frac{dU}{dx}$$



## Potential Energy Diagram for an ideal spring



## Vertical mass-on-spring



Treat the **equilibrium** length of the spring as if it is the **unstretched** length of the spring. Then, you can **neglect the gravitational force** on the object.

The spring becomes a compressible spring.



## Example

A spring has a stiffness 100 N/m. You stretch it 20 cm from its unstretched position. How much work did you do on the spring? If you stretch it an additional 20 cm, how much *additional* work did you do on the spring?



## Example



You hang 0.25 kg on a spring of stiffness 10 N/m. You pull it down 0.05 m from its equilibrium position and release it from rest. How fast is it moving when it reaches the equilibrium position?



### Example

You use a compressible spring in a dart gun to shoot a dart upward. The dart has a mass of 10 grams, the spring has a stiffness of 200 N/m, and you initially compress the spring 0.05 m. If you release it from rest, how fast is the dart moving when it leaves the spring?



### Example

For the dart in the previous question, how high will it go (assuming no air resistance)?



### Example

You build a model of a bungee jumper using a spring of stiffness 5 N/m and a mass of 0.2 kg. The fixed end of the spring is at  $y=0$ , and the spring is 10 cm long. You release the object at  $y=0$ . How far does the object fall before being “jerked” upward?



### Example

An object with mass attached to an ideal spring is called a simple harmonic oscillator. Suppose you set up a simple harmonic oscillator in lab using an object of mass 0.25 kg and a spring of stiffness 20 N/m. You pull the object back from its equilibrium position a distance of 0.1 m and release it from rest. As a result, the object oscillates in simple harmonic motion.

- What is the energy of the system when the spring is maximally stretched?
- What is the energy of the system when the object is at the equilibrium position?
- What is the energy of the system when the object is at  $x=1/2A$ .
- If you pull back the object twice as far as in the first experiment, what happens to the energy of the system and the frequency of oscillation compared to the first experiment?



### Poll

A simple harmonic oscillator of a mass  $m$  on a spring of stiffness  $k$  oscillates with an amplitude  $A$  and frequency  $\omega$ . If you double the **mass**  $m$ , the **energy** of the system

- Increases by factor of 2
- Decreases by factor of 1/2
- Increases by factor of 4
- Decreases by factor of 1/4
- Increases by factor  $\sqrt{2}$
- Decreases by factor  $1/\sqrt{2}$
- Stays the same.



### Poll

A simple harmonic oscillator of a mass  $m$  on a spring of stiffness  $k$  oscillates with an amplitude  $A$  and frequency  $\omega$ . If you double the **stiffness**  $k$ , the **energy** of the system

- Increases by factor of 2
- Decreases by factor of 1/2
- Increases by factor of 4
- Decreases by factor of 1/4
- Increases by factor  $\sqrt{2}$
- Decreases by factor  $1/\sqrt{2}$
- Stays the same.



## Poll

A simple harmonic oscillator of a mass  $m$  on a spring of stiffness  $k$  oscillates with an amplitude  $A$  and frequency  $\omega$ . If you double the **amplitude**  $A$ , the **energy** of the system

1. Increases by factor of 2
2. Decreases by factor of 1/2
3. Increases by factor of 4
4. Decreases by factor of 1/4
5. Increases by factor  $\sqrt{2}$
6. Decreases by factor  $1/\sqrt{2}$
7. Stays the same.



## Poll

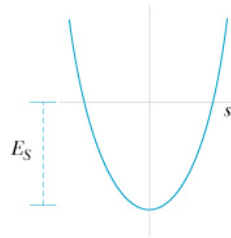
A simple harmonic oscillator of a mass  $m$  on a spring of stiffness  $k$  oscillates with an amplitude  $A$  and frequency  $\omega$ . If you double the **frequency**  $\omega$ , the **energy** of the system

1. Increases by factor of 2
2. Decreases by factor of 1/2
3. Increases by factor of 4
4. Decreases by factor of 1/4
5. Increases by factor  $\sqrt{2}$
6. Decreases by factor  $1/\sqrt{2}$
7. Stays the same.

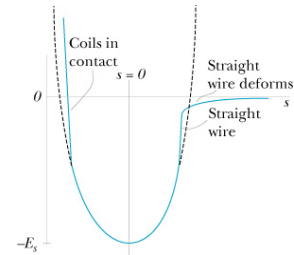


## Potential Energy for a real spring

$$U_{elas} = \frac{1}{2}ks^2 - E_s$$



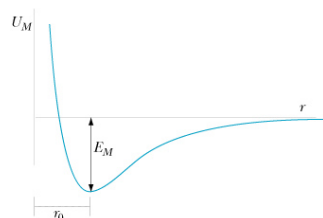
## Problems with an ideal spring...



## Modeling a chemical bond

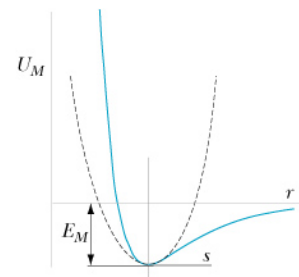
Morse Potential

$$U_M = E_M \left( 1 - e^{-\alpha(r-r_{eq})} \right)^2 - E_M$$



For small oscillations, anything is a simple harmonic oscillator...

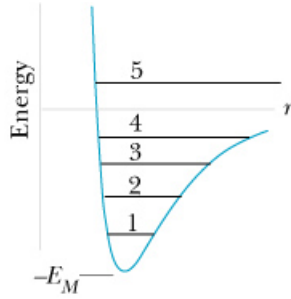
For small oscillations about the equilibrium position, a diatomic molecule can be treated a harmonic oscillator.





### Poll

Five different total energies for a diatomic molecule are shown. For which energies is the molecule bound?

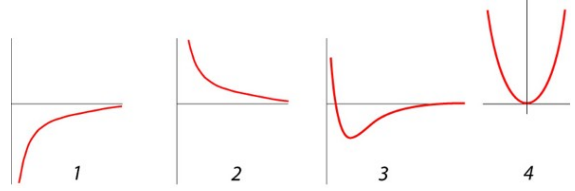


- 6. 1 - 4
- 7. All of them
- 8. None of them



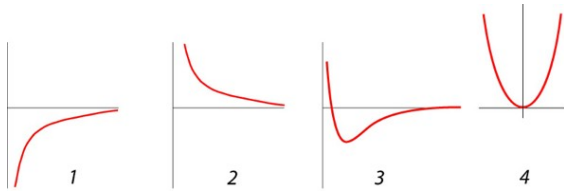
### Poll

Which of these potential energy diagrams is for two interacting electrons?



### Poll

Which of these potential energy diagrams is for two bodies of significant mass?



### Poll

Which of these potential energy diagrams is for a spring?

