PHY 221: Wavefunction, Wave Superposition, Standing Waves on a String

Objective

Write a mathematical function to describe the wave. Describe a transverse wave and a longitudinal wave. Describe frequency, wavelength, and amplitude for a sinusoidal transverse wave and a sinusoidal longitudinal wave. Describe the term *superposition* of waves. Describe what is meant by *standing waves* and what is required for the production of standing waves.

Period, frequency, amplitude, wavelength, and wave speed.

We will investigate a transverse wave on a string and attempt to completely describe that wave by a function $y(x, t)$. This function tells us the vertical position of a point on the string at the horizontal position x at some instant of time t. Note that the same function tells us the displacement of the position of a point on the string from equilibrium if equilibrium is defined to be $y = 0$.

Go to our course web site, click the link to `Java Simulations`, and then click the link to the simulation `Frequency, period, and speed of a transverse wave on a string`. The string is modeled as balls connected by springs, though the springs are not shown. The type of wave that you see is called a *transverse wave*. Describe in words the motion of a piece of string when a transverse wave travels on a string.

Click and hold the left mouse button. Note that as you move the cursor around the animation window, the (x,y) coordinates of the mouse cursor are displayed in the animation window. The clock reading is displayed in the upper left corner. Use this technique to measure position of a point on the string at any clock reading.

What is the period T of the motion of the green piece of string?

This is also called the *period* T of the wave.

What is the frequency of the motion of the green piece of string, in cycles per second (Hz)?

This is also called the *frequency* f of the wave and is related to period by

$$f = \frac{1}{T} \tag{1}$$

If you convert the units of frequency to radians per second, then it is called angular frequency. The conversion factor is 2π rad per cycle, so

$$\omega = 2\pi f \tag{2}$$

What is the amplitude of the oscillation of the green piece?

This is called the *amplitude* of the wave.

What is the horizontal distance between crests of the wave?

This is called the *wavelength* λ of the wave.

Measure the time interval for a wave crest to travel a distance equal to one wavelength. Do this by pausing the animation at some instant and measuring the x-position and clock reading of a wave crest. Advance the animation for a time interval equal to the period and measure the x-position of the same wave crest. Use these measurements to determine the distance traveled by the wave crest. What do you notice about the result?

This gives an alternative way to define the wavelength of a wave–it is the distance traveled by a wave crest (or wave trough) in a time interval of one period.

Measure the speed of the wave by calculating the distance traveled divided by the time interval.

In terms of wavelength λ and period T, write a formula for the speed v of the wave. (Use no numbers; just write a formula for wave speed in terms of wavelength and period.

Substitute $f = 1/T$ into the equation for wave speed to write the wave speed in terms of wavelength and frequency.

The above equation for wave speed in terms of wavelength and frequency should be memorized.

Wavefunction

Now, we need an equation that will tell us the vertical position of any piece of the string at the location x at any instant of time t. Certainly, amplitude, frequency, and wavelength will all be important. The other important variable is the *phase* which is a constant that is determined by the y-position of the string at $x = 0$ when the clock starts at $t = 0$.

The equation is

$$y = A\sin(kx - \omega t + \phi) \tag{3}$$

where A is the amplitude, k is called the wavenumber and is equal to $2\pi/\lambda$ and ω is the angular frequency. In terms of wavelength and frequency we can write the function as

$$y = A\sin(\frac{2\pi}{\lambda}x - 2\pi ft + \phi) \tag{4}$$

For the wave that you studied in the first part of this activity, use its wavelength and frequency to write an equation for y as a function of x and t.

Go back to the course web site, and again click the link to Java Simulations. Click the link to `Wavefunction (for` $\phi = 0$`)`. Measure the x-position of the red piece of string. Write a function y(t) for the red piece of string by substituting the x-value of the red piece and $\phi = 0$ into the equation for y(x,t) that you developed above. (The wavelength and frequency of the given wave is the same as the previous wave that you studied.) Enter your function into the box and click the Start Animation link. Does your y(t) function match the actual y vs. t graph for the red piece? Write y(t) below.

Types of waves

Go back to the course web site and view the Java Simulations. Click the link to the simulation **`Frequency, period, and speed of a longitudinal wave on a spring`**. The wave that you now see is called a longitudinal wave. How is it different from a transverse wave?

It's possible for a wave to have both a longitudinal and transverse component. Geologists use waves traveling in Earth's crust to figure out what Earth is made of or to find oil deposits or magma or whatever. Seismic waves have both a transverse component (called a S wave by geologists) and a longitudinal component (called a P wave by geologists).

For the longitudinal wave in this simulation, measure the amplitude, wavelength, frequency, and period of the wave. You should be able to use measurements directly from the animation window to measure and/or calculate these quantities. The graph of Δx vs. t for the green piece may also be helpful.

Write a general equation for the x-displacement of a piece of the string as a function of its equilibrium position x along the string and time t.

Wave Interactions

When there is more than one wave (or pulse) traveling in the same medium, they interact. The resulting wave is the sum of the two waves. The sum of two or more waves is called the *superposition* of the waves, and the interaction of waves in general is called *interference*.

1. Go back to our course web site and Java Simulations. Click the link to the `Superposition of Waves` simulation.

2. Click the link for Animation 1. In this simulation, you will see two identically shaped wave pulses, one traveling to the right and the other one traveling to the left. The bottom pane of the simulation shows the sum of the two waves. Play the simulation and view the results. You may stop the simulation and step it forward or backward at any moment to see the results.

Measure the height of each of the pulses. What do you predict will be the maximum height of the sum of the two pulses when they exactly coincide?

Stop the simulation when the two pulses are at the same x-position. Measure the height of the superposition and compare to your prediction.

When the superposition of two waves has an amplitude greater than the individual waves, we say that they *constructively* interfere.

Suppose that the pulse traveling to the left is inverted so that it is upside-down. What do you predict would be the height of the superposition when the pulses are at the same x-position?

Click the link to Animation 2 and compare the result with your prediction.

When the superposition of two waves has an amplitude less than the individual waves, we say that they *destructively* interfere.

When two waves interfere, does it destroy the individual waves?

3. Now, let's look at interfering transverse waves. Click the link to Animation 3. These waves are identical in every way. They have the same wavelength, same frequency, same amplitude, and same direction of propagation.
4. Measure the y-position of each wave at $t = 0$ and $x = 1$.
5. Measure the y-position of the superposition of the two waves at the same values of x and t.

Does $y = y_1 + y_2$? (y is the y-position of the superposition and y_1 and y_2 are the y-positions of each of the two waves.)

6. Now, let's look at interfering transverse waves where one wave is phase shifted compared to the other wave. Click the link to Animation 4. These waves are identical in every way EXCEPT one wave is phase shifted 180° (or π radians) relative to the other wave. This makes the crest of one wave line up with the trough of the other wave.
7. Measure the y-position of the superposition of the two waves at the same values of x and t.
8. Measure the y-position of the superposition of the two waves at the same values of x and t (this should be obvious and doesn't really require a measurement).

Does $y = y_1 + y_2$? (y is the y-position of the superposition and y_1 and y_2 are the y-positions of each of the two waves.)

9. Now, take time to play with adding waves of different amplitude, different wavelength, different frequency, or different phase.

   NOTE THAT YOU SHOULD KEEP THE SPEED OF THE WAVES THE SAME. That is, if you double the wavelength, you should change the frequency by $\frac{1}{2}$ so that $v = \lambda f$ is constant. We'll learn later why this is the case.

What "part" of the wavefunction affects whether the wave moves to the right or to the left? Experiment with the wavefunctions to see if you can get the wave to move to the left.

Standing Waves

What happens when you add two waves that are identical in every way EXCEPT that they travel in opposite directions?

(a) In the box for y_1, enter the function $2 * \sin(2 * pi/8 * x - 2 * pi * 0.5 * t)$.

(b) In the box for y_2, enter the function $2 * \sin(2 * pi/8 * x + 2 * pi * 0.5 * t)$

Describe the superposition of the two waves.

This is called a *standing wave*.

Note that each point of the standing wave is oscillating in simple harmonic motion. But what is different about the simple harmonic motion ($y(t)$) of each point on the standing wave?

Write an equation for y as a function of time for the point on the string at x=0.

Write an equation for y as a function of time for the point on the string at x=1.

Write an equation for y as a function of time for the point on the string at x=2.

We can write a general equation for the vertical displacement y of a piece of string at some horizontal position x on the string for a standing wave at time t. It is given by superposition of the two traveling waves that are identical, except for direction of motion.

$$y = y_1 + y_2 = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

There is a trig identity that says:

$$\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$$

Therefore, for a standing wave,

$$y = 2A\sin(kx)\cos(\omega t) \quad (5)$$

Note how unbelievably clear this equation is! Each point oscillates up and down with an amplitude that depends on where it is on the string. For a point at position x, its amplitude of oscillation is $2A\sin(kx)$ which is exactly what we noticed.

Those points where the amplitude of oscillation is zero are called *nodes* and those points where the amplitude of oscillation is a maximum are called *antinodes*.