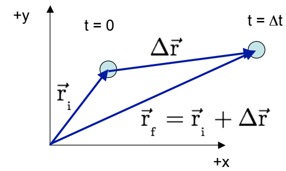
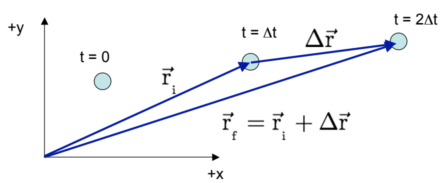
CH-01-3: Predicting Position

Predicting Position - first time step



The position of the object at t=Δt is its initial position plus its displacement.

Predicting Position - second time step

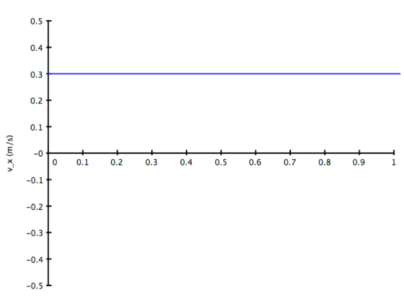


The position of the object at t=2Δt is its initial position (at t=Δt) plus its displacement.

Simple Example

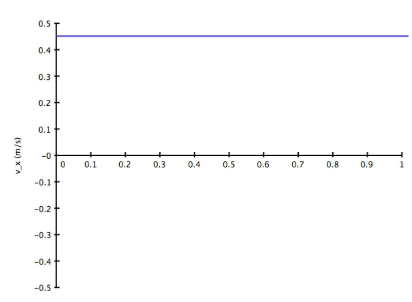
A steel ball is at x=0.20 m at t=0. Its velocity is constant and equal to <0.3,0,0> m/s. Using 0.1 s time steps, what is the position of the ball at t=0.5 s. Use the position update method.

Area under a x-velocity vs. time graph



Simple Example

What is the displacement of the object between t = 0.5 s and t = 0.6 s? Sketch this on the x-velocity vs t graph.



Simulating uniform motion with VPython

1. Create a ball
2. Give it an initial position and velocity.
3. Define a time step dt.
4. Create a while loop.
5. Update the ball's position in the while loop over and over and over again.

Newton's First Law

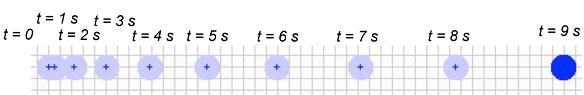
A reference frame in which Newton's first law is valid is called an **inertial reference frame**.

Newton's laws are not valid in a non-inertial reference frame.

If an object (or system) is isolated from interactions, its velocity should be constant. If it is not, then this is a non-inertial reference frame.

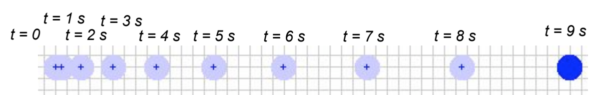
Simulation

Non-uniform motion



Measure the average velocity between t=5 and t=6 s.
Use this to predict the position at t=7 s.

Instantaneous velocity



Velocity at an instant of time (i.e. a single clock reading) is ***instantaneous velocity***.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

Poll

At t=15 s two bees are observed to be at the position < 2, 4, 0> m. Bee #1 flies in a straight line with constant speed and arrives at position <3, 3.5, 0 > m at t=15.5 s. Bee #2 buzzes around, repeatedly changing speed and direction, sometimes going quickly and other times just hovering in the air, but it also arrives at position < 3, 3.5, 0 > m at t=15.5 s.

Which statement about their average velocities is correct?

1) The magnitude of Bee #1's average velocity is greater.
2) The magnitude of Bee #2's average velocity is greater.
3) The two bees have the same velocity at all times.
4) The two bees have the same average velocity although their velocity at a given time may not be the same.

Poll

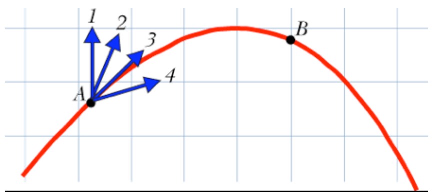
At t=12.18 s, a ball's position is < 20, 8, -12> m, and the ball's velocity is < 9, -4, 6 > m/s.

What is the (vector) position of the ball at t=12.21 s? Assume that the ball's velocity does not change significantly during this short time interval.

1) 24.75 m
2) < 20.27, 7.88, -11.82 > m
3) < 0.27, -0.12, 0.18 > m
4) < 129.62, -40.72, 61.08 > m
5) < 129.89, -40.84, 61.26 > m

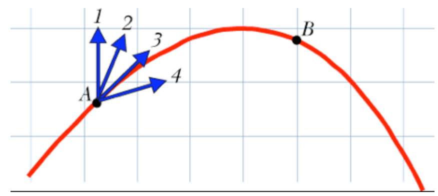
Poll

A ball travels through the air. Part of its trajectory is shown in red. Which arrow best represents the direction of the average velocity of the ball as it travels from location A to location B?

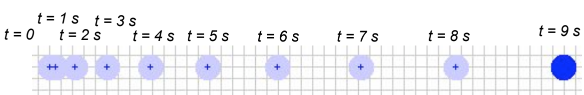


Poll

A ball travels through the air. Part of its trajectory is shown in red. Which arrow best represents the direction of the instantaneous velocity of the ball at location A?



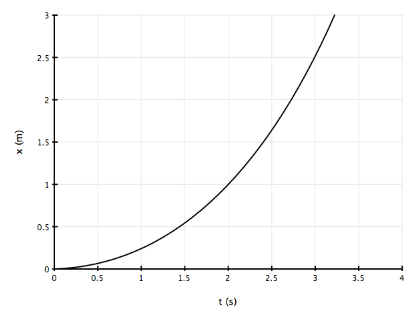
Non-uniform motion - constant force



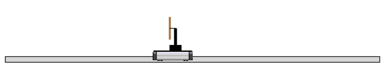
For example, if you fire a single thruster on our VPython Spaceship, the motion of the spaceship will look like the picture above.

What do you notice about the displacement between successive images?

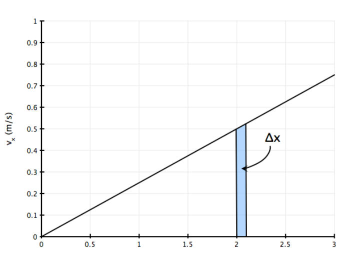
constant force -- x vs. t graph

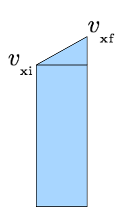


constant net force - v_x vs t graph

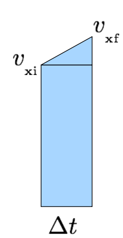


fan cart





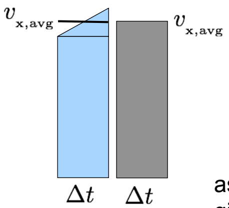
area under the v_x vs. t graph



$$
\begin{aligned}
\Delta x &= v_{xi}\Delta t + \frac{1}{2}\Delta t(v_{xf} - v_{xi}) \\
&= v_{xi}\Delta t + \frac{1}{2}\Delta t v_{xf} - \frac{1}{2}\Delta t v_{xi} \\
&= \frac{1}{2}v_{xf}\Delta t + \frac{1}{2}v_{xi}\Delta t \\
&= \underbrace{\left(\frac{v_{xf} + v_{xi}}{2}\right)}_{v_{x,avg}}\Delta t
\end{aligned}
$$

mean velocity

$$\begin{aligned}\Delta x &= v_{xi}\Delta t + \frac{1}{2}\Delta t(v_{xf} - v_{xi}) \\ &= v_{xi}\Delta t + \frac{1}{2}\Delta t v_{xf} - \frac{1}{2}\Delta t v_{xi} \\ &= \frac{1}{2}v_{xf}\Delta t + \frac{1}{2}v_{xi}\Delta t \\ &= \underbrace{\left(\frac{v_{xf} + v_{xi}}{2}\right)}_{v_{x,avg}}\Delta t\end{aligned}$$

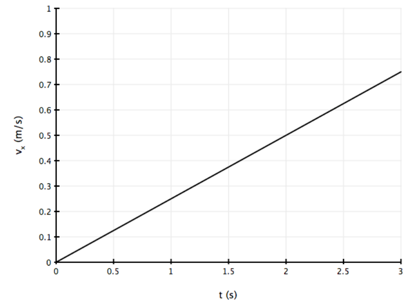


assuming a constant velocity of v_{avg} gives the same result.

Predicting Position

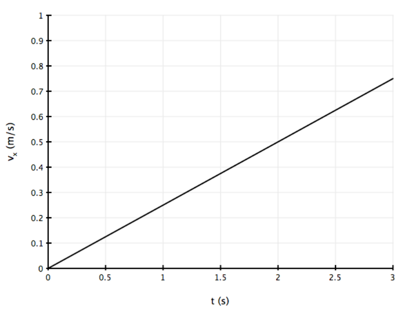
$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$$

$$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$$



Simple Example

If the spaceship in a simulation is at x=1.0 m at t = 1.0 s and its thruster continuously fires in the +x direction, what will be its position at t = 3.0 s?



LAB -- Video Analysis of a Fan Cart