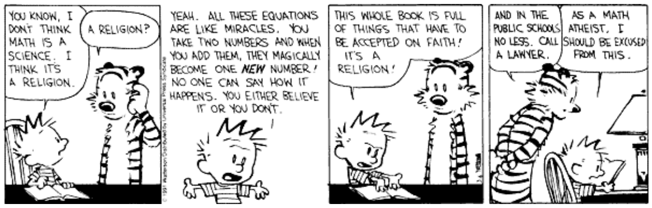
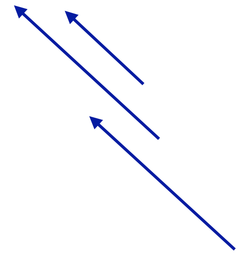


Chapter 1

Vectors

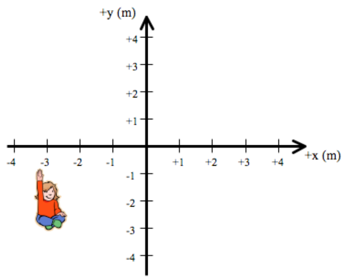
Vector Definition

- A quantity that has two properties: ***magnitude*** and ***direction***
- It is represented by an arrow; visually the length represents magnitude
- It is typically drawn on a coordinate system



Example 1 (Position)

- What is the girl's position vector?



Example 2 (Position)

- A car sits at the position

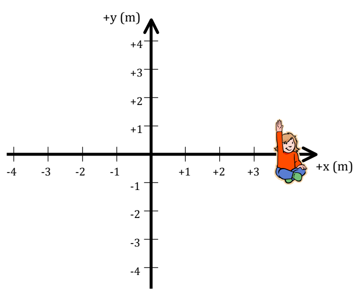
  < 10 , -4 , 0 > m

  Sketch its position vector.

Poll

What is the girl's position vector? Sketch it and write it in component form.

A. < 0, 4, 0 > m
B. < 4, 0, 0> m
C. < 2, 4, 0> m
D. < 4, -2, 0> m
E. None of the above.



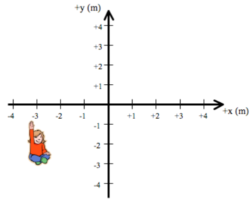
Activity: What is the position of the center of your tabletop?

- Define the NE corner of the lab floor to be the origin.
- Define the +x axis to be along the whiteboard.
- Define the +y axis to be perpendicular to the whiteboard.
- Define the +z axis to be upward, toward the ceiling.

You have 10 minutes.

Vector Magnitude

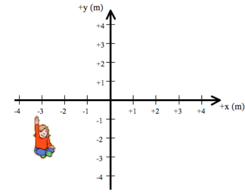
- Represented by the length of the vector.
- Use the Pythagorean Theorem



$$|\vec{r}| = \sqrt{r_x^2 + r_y^2 + r_z^2}$$

Vector Direction

- Represented by a ***unit vector*** that has a magnitude of 1 and points in the same direction.



$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

Poll

What is the magnitude of the vector < 3, 5, −2 >?

A. 5.48
B. 6.16
C. 6.00
D. 30.00
E. 38.00

Poll

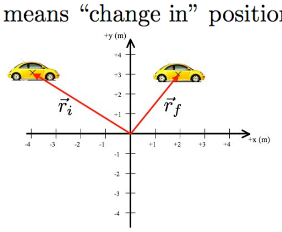
What is the unit vector in the direction of the vector < 3, 5, −2 >?

1. < 3, 5, −2 >
2. < 1, 1, −1 >
3. < 0.49, 0.81, 0.32 >
4. < 0.49, 0.81, −0.32 >
5. < 0.3, 0.5, −0.2 >

Displacement

$\Delta\vec{r}$ means "change in" position

- ***Displacement*** is the vector **from** an ***initial*** position vector **to** a ***final*** position vector.
- Displacement tells you direction and distance and object moved.
- Displacement is the ***"change in position"*** of an object.



Subtracting Vectors Algebraically

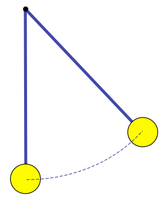
$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

Example: You walk from the position < 1.5 , 1 , 0 > m to the position < 5 , 3 , 1 > m. What is your displacement?

Subtracting Vectors Pictorially

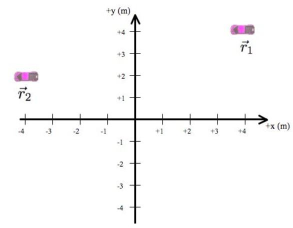
- The ***change in*** a vector points **from** the **initial** vector **to** the **final** vector.
- To sketch displacement, draw an arrow from the initial position to the final position.

A pendulum swings from its highest points to its lowest point. Sketch its initial position, final position, and displacement. (Define the origin to be at the pivot.)



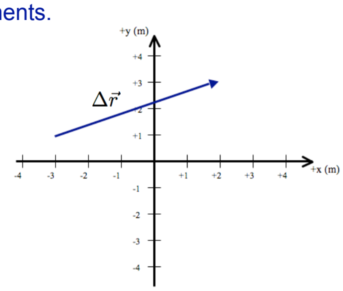
Vector Components

- ***Vector components*** are measured from the tail of the vector.
- Draw a right triangle.
- Measure the x and y components



Example (Vector Components)

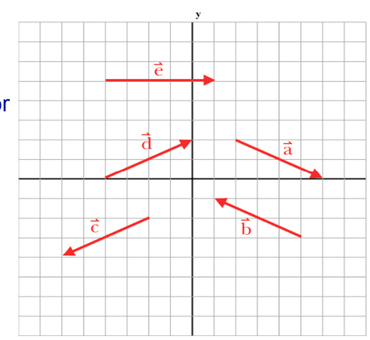
What are the components of this displacement vector? Sketch the x and y components.



Poll

Which of these arrows represents the vector < -4, 2, 0 >?

A. $\vec{a}$
B. $\vec{b}$
C. $\vec{c}$
D. $\vec{d}$
E. $\vec{e}$

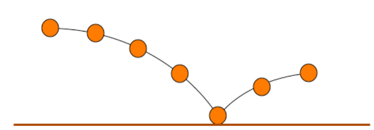


Signs of Displacement Components

- What does it mean if the displacement of an object has a ***positive*** x-component?
- or ***negative*** x component
- or positive y component
- or negative y component?

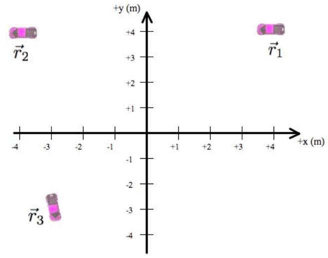
Total Displacement

- Successive displacements determines an object's path.
- The ***total displacement*** is the sum of the individual displacements.



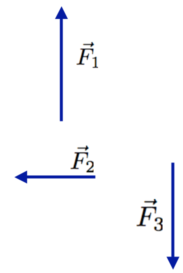
Adding Vectors Pictorially

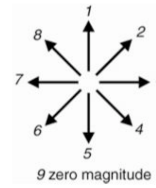
- Place vectors head-to-tail, one after the other. The sum of the vectors is drawn from the tail of the first vector to the head of the last vector.
- This is called the ***resultant***.



Poll

The following forces act on an object. If you add the force vectors, what is the direction of the resultant?





Adding Vectors Algebraically

While giving a tour, a HPU ambassador walks < -90 , -30 , 0 > m from Wrenn and from this point walks < -60 , 50, 0 > m. If the +y direction is North and the +x direction is East. What is the total displacement of the ambassador? How far is the ambassador from her starting point?

$$\Delta\vec{r}_{total} = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \cdots$$

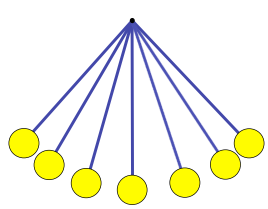
Poll

A football is at the location < 20, 30, 2 > m and is then displaced < 30, 40, 10 > m. What is its new position?

Path of an Object

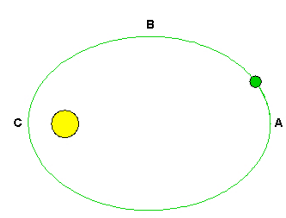
- By measuring small displacements, one can map the path of an object.
- An object's ***direction of motion*** is always tangent to the path because a small displacement vector is tangent to the path.

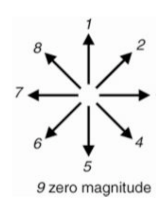
A pendulum swings back and forth. What is the pendulum's direction of motion at the lowest point of its swing, as it travels from right to left?



Poll

A comet orbits Sun as shown below. What is the ***direction of motion*** of the comet at point A?

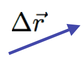




Multiplying a Vector by a Scalar

- A ***scalar*** is quantity represented solely by a number.
- A scalar can be positive or negative.
- Multiplying a vector by a scalar changes the ***magnitude*** of the vector.
- Multiplying by -1 ***reverses the direction*** of the vector.

Sketch $2\Delta\vec{r}$



Sketch $-3\Delta\vec{r}$

Example

A runner has a displacement < 100, 50, 0> m. Another runner has a displacement of 2 times the first runner. What is her displacement?

Poll

An HPU statue is at the position < 30, -40, 0> m relative to the front door of Congdon Hall. If you walk exactly halfway toward this statue, what is your position?

(A) < 60, -80, 0 > m

(B) < 30, -40, 0> m

(C) < 15, -40, 0> m

(D) < 30, -20, 0> m

(E) None of the above.

Angles

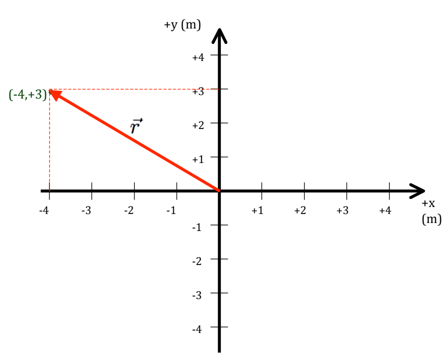
- Sometimes a vector is described in terms of ***magnitude*** and ***angle*** with respect to the coordinate axes.
- Use trig functions to calculate angles.
- Memorize the definitions of ***cosine***, ***sine***, and ***tangent***.

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

2-D Vector



Use Angles and Magnitude to Determine the Vector

- Sketch a picture.
- Use trig functions to calculate vector components.
- Write your vector in component form.

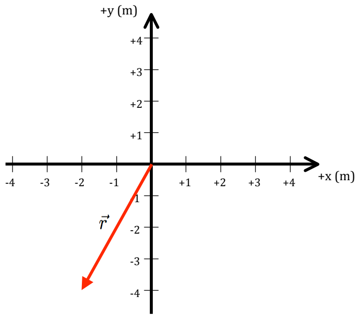
$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

Example

Sketch a right triangle for the vector shown. Find the angles within the right triangle and find the angles the vector makes with the +x and +y axes.



Example

A child pulls a sled with a 2-m long rope that makes an angle of 30° with respect to the ground. (1) How high is the child's hand above the point where the rope is attached to the sled? (2) How far is the child in front of the sled? (3) What is the position of the child's hand with respect to where the rope is attached to the sled?

Poll

A swimmer dives from a 1.5-m high springboard. He hits the water at a horizontal distance of 4.0 m from the end of the springboard. What is the magnitude of his displacement between the end of the board and the water, and what angle does it make with the water?

(A) 4.3 m, 21°

(B) 4.3 m, 69°

(C) 4.5 m, 43°

(D) 4.5 m, 47°

(E) None of the above

Poll

You pull a 1-m long pendulum to the right until it hangs at an angle of 15° from vertical. Define the +x axis to be to the right and the +y axis to be vertical and ***downward***, with the origin at the pivot of the pendulum. What is the final position of the pendulum?

(A) < -0.97, 0.26, 0> m

(B) < -0.26, 0.97, 0> m

(C) < -0.85, 0.15, 0> m

(D) < -0.92, 0.21, 0> m

(E) None of the above

Direction Cosines

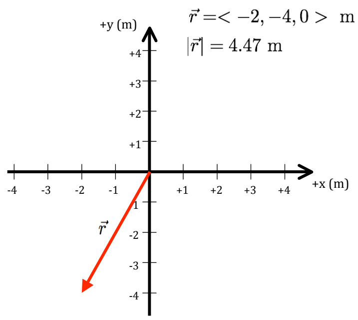
$$\cos\theta_x = \frac{r_x}{|\vec{r}|}$$

$$\cos\theta_y = \frac{r_y}{|\vec{r}|}$$

$$\cos\theta_z = \frac{r_z}{|\vec{r}|}$$

$\vec{r} = < -2, -4, 0 >$ m

$|\vec{r}| = 4.47$ m



Unit Vector and Direction Cosines

- The unit vector is equal to the direction cosines.
- Use the unit vector to calculate angles with respect to each axis.
- Draw a picture to get the angles correct.

What are the direction cosines (and angles with respect to each axis) for the vector <6,-3,-2>?