PHY 221 LAB 04-4: Standing Waves on a String

In this experiment, standing waves will be produced by an oscillator that moves a string up and down sinusoidally. The resulting wave travels down the string and reflects off a fixed node at the end. The traveling waves interfere. If the tension in the string is just right so that the speed of the wave has a wavelength that is a half-integer multiple of the length of the string, then a standing wave will result.

Harmonics

A standing wave exists when two waves of the same speed travel in opposite directions and interfere. If the wavelength of the wave is such that there is a node at the beginning and end of the string. The longest wavelength for this to occur is when

$$\lambda = 2L \tag{1}$$

which is shown in Figure 1.

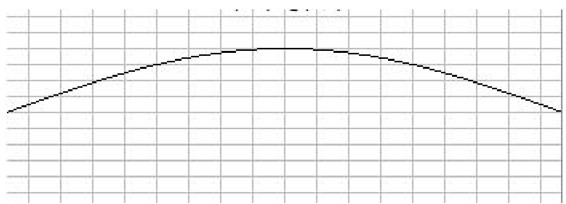

Figure 1: First harmonic, n=1 (the fundamental).

If the wavelength is shorter, the next possible standing wave is for

$$\lambda = L \tag{2}$$

which is shown in Figure 2.

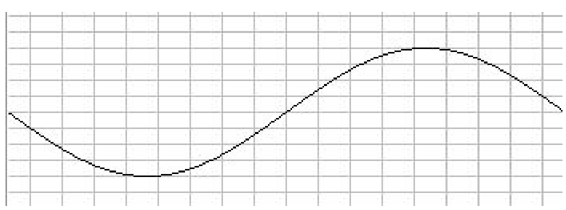

Figure 2: Second harmonic, n=2.

If the wavelength is shorter by half a wavelength, the next possible standing wave is for

$$\lambda = \frac{2}{3}L \tag{3}$$

which is shown in Figure 3.
Again, shorten the wavelength by half a wavelength, and you'll get

$$\lambda = \frac{1}{2}L \tag{4}$$

which is shown in Figure 4.
We can write a general equation relating wavelength of the standing wave to the length L of the string.

$$\lambda = \frac{2L}{n} \tag{5}$$

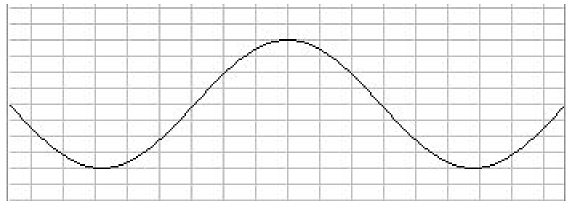

Figure 3: Third harmonic, n=3.

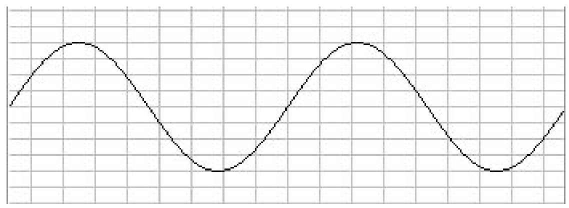

Figure 4: Fourth harmonic, n=4.

where $n = 1, 2, 3, 4, ...$ and each integer is referred to as a *harmonic*. The longest wavelength occurs for $n = 1$, the *first harmonic* which is called the *fundamental*. Higher harmonics are referred to as *overtones*. The second harmonic is called the *first overtone*.

Note that the number of nodes in each case is

$$\#\text{ of nodes} = n + 1 \tag{6}$$

Speed of a wave on a string

For any wave, its speed is related to its wavelength and frequency.

$$v = \lambda f \tag{7}$$

The wave speed also depends on the medium it travels in. For a wave on a string, its speed depends on the tension in the string and the linear density of the string μ. (Linear density is its mass per unit length, so $\mu = m/L$). The speed of a wave on a string is

$$v = \sqrt{\frac{T}{\mu}} \tag{8}$$

Stringed Instruments

Equating the above two equations, then

$$\lambda f = \sqrt{\frac{T}{\mu}} \tag{9}$$

For a standing wave on a string, substitute $\lambda = \frac{2L}{n}$ and solve for the frequency of the standing wave in terms of tension and linear density of the string, as well as the harmonic n.

$$\left(\frac{2L}{n}\right) f = \sqrt{\frac{T}{\mu}} \tag{10}$$

$$f = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \tag{11}$$

Suppose that a standing wave on a string on an instrument (like piano or violin or guitar, etc.) vibrates with the fundamental frequency ($n = 1$). Then,

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \tag{12}$$

To *increase* the frequency, you can:

- decrease the length of the string (by pressing down on the string).
- increase the tension (by tightening a screw, or "key").
- decrease the mass per unit length (by using a less dense string, typically a thinner string or one made of a different material)

What can you do to decrease the fundamental frequency on a string?

Sound from stringed instruments

Any periodic waveform can be written as a sum of cosine and sine functions. This is called a *Fourier Series.*

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\frac{2\pi n}{T}t) + \sum_{n=1}^{\infty} b_n \sin(\frac{2\pi n}{T}t) \tag{13}$$

The constants a_0, a_n and b_n are found by integrating the functions below.

$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} f(t)dt \tag{14}$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(\frac{2\pi n}{T}t)dt \tag{15}$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin(\frac{2\pi n}{T}t)dt \tag{16}$$

$$\tag{17}$$

Some examples are shown in Figures 5 – 7. In all cases, a period $T = 2\pi$ is used.

The important idea to remember is that any periodic wave can be approximated as a Fourier Series. When a sound is played on an instrument, perhaps by plucking a string (e.g. guitar) or hammering the string (e.g. piano), a wave is produced that is not a simple sine wave. But rather, it is made up of many

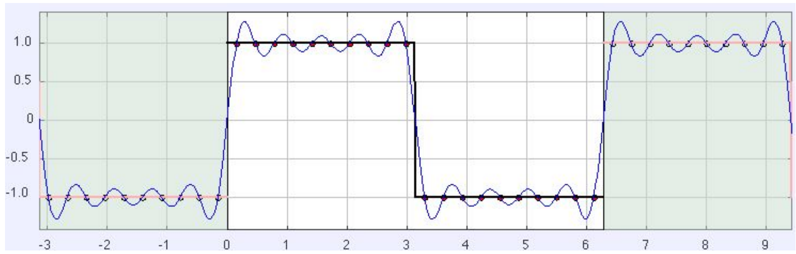

Figure 5: A square wave approximated as a Fourier Series.

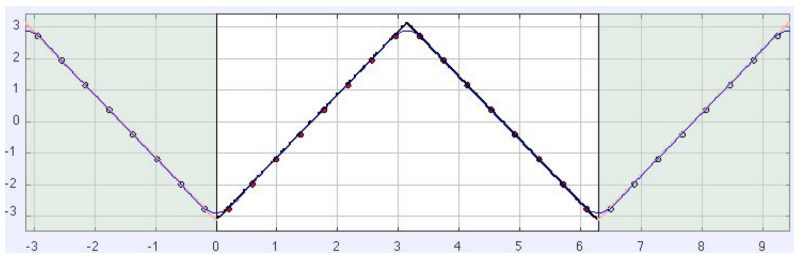

Figure 6: A triangle wave approximated as a Fourier Series.

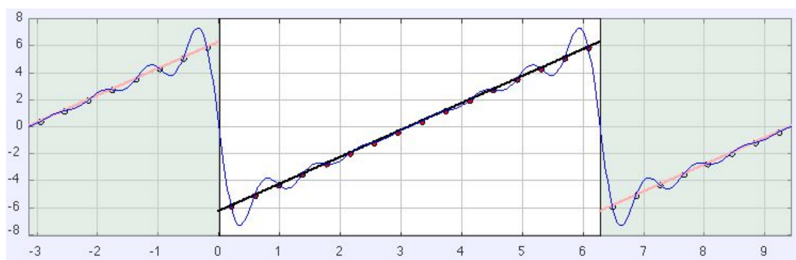

Figure 7: A sawtooth wave approximated as a Fourier Series.

sine waves (i.e. a Fourier Series). Some of the sine waves have large amplitude (the fundamental) and other sine waves have lower amplitude (the overtones).

A middle C on piano and the same note on a guitar have the same fundamental, but they have overtones with different amplitudes. As a result, the "mixture" of sine waves is different for the two instruments even though the fundamental frequencies are the same.

By doing a *Fourier transform* using a computer, you can determine the amplitudes (a_n and b_n) of the harmonics for the wave that produced the sound for the instrument. The image below shows the amplitudes of the harmonics that make up the square wave in Figure 5. Notice that the fundamental (n=1) is the strongest. The only harmonics are the odd harmonics, and they decrease in amplitude.

Experiment–Standing Waves on a String

1. Set up an oscillator using a ringstand attached to one end of the table.

2. On the other end of the table, set up a ringstand with a pulley.

3. Attach one end of a long string to the oscillator and run the other end of the string over the pulley. Be sure that the pulley can freely rotate.

4. Hang a bucket on the end of the string.

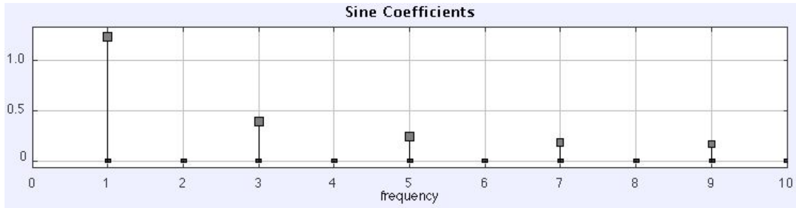

Figure 8: The amplitudes of the harmonics that make up the Fourier Series for the square wave.

5. Measure the length L of the string from the oscillator to the top edge of the pulley, above the axle.
6. Plug in the oscillator and begin to add BBs to the bucket.
7. Add BBs at a slow rate until you begin to see what looks like a standing wave. Add BBs one at a time until the standing wave has its greatest amplitude and is very stable (meaning that it doesn't appear and disappear or change amplitude). An example of a standing wave is shown below.

Figure 9:

8. Record the number of nodes that you see for the standing wave (including the endpoints). Calculate the wavelength of the wave.

   Create a datatable for your data and calculations, similar to the one shown below.

n	# of nodes	λ (m)	mass (m)	Tension (N)

9. Measure the mass of the bucket and BBs. Calculate the gravitational force on the bucket. This is the tension in the string.
10. Add additional BBs until you see a different wavelength standing wave. Record the data for this standing wave.
11. Repeat these measurements for a total of 5 harmonics. Be sure to get one data point for the second harmonic. It is best not to try for the first harmonic since the apparatus may not be strong enough for the high tension required.

Sketch a picture of each of the standing waves that you saw. Write the wavelength of the standing wave in each case.

12. The frequency of the oscillator is 120 Hz. Calculate the speed of the wave for each of the standing waves that you studied. (Just add a new column to your data table.)

Graph the speed of the wave v vs. the tension T. Use Equation (7) to identify a function for the best-fit curve. Fit the curve to your data. Record the function for the best-fit curve.

The constant of your best-fit curve to v vs. T is equal to $1/\sqrt{\mu}$ where μ is the linear density, or mass per unit length, of the string. Using your equation, calculate the linear mass density of the string.

Experiment–Fourier Transform of Sound

In this experiment, you are going to measure the harmonics of a sound wave using a Fourier Transform.

1. Connect a microphone to the LabPro data acquisition interface.
2. Open Logger Pro.
3. You will see a graph of sound intensity vs. time. The sound intensity is proportional to the square of the amplitude of the sound wave.

4. Say a continuous sound like "oooooooooooo" into the microphone while your lab partner clicks the $\boxed{\text{Collect}}$ button. Continue saying the sound until you see a graph.
5. Go to the menu `Insert`→`Additional Graph`→`FFT`.
6. The resulting graph shows the amplitudes (y-axis) of various frequencies (x-axis) of the sine and cosine waves that make up the sound that you produced. These frequencies are called harmonics.
7. Using your mouse, measure the frequency of the greatest amplitude harmonic. (You can go to the menu `Analyze`→`Examine` or click the Examine icon on the toolbar to more easily make your measurement.) Record your result.
8. Also, measure and record the second and third greatest amplitude harmonics.
9. Now, make a different sound, like "ahhhhhhhhh" for example. Again, measure the frequencies of the top three greatest amplitude harmonics.
10. Ask a guitar player in the class to play a note for you. Measure the frequencies for the top three harmonics of the note that he plays.
11. Measure and record the top three harmonics for a different note.

Application

1. In the experiment on standing waves on a string, the frequency of the oscillator stayed the same. As you increased the tension, did the harmonic (n) increase or decrease? Did the wavelength of the standing waves increase or decrease? Did the speed of a wave on the string increase or decrease?
2. On a guitar string (or piano string or violin string), the predominant frequency that you hear is due to the fundamental (n=1). So, this stays constant. As you change the tension, does the wavelength increase or decrease? Does the frequency increase or decrease? Does the speed of a wave on the string increase or decrease?
3. Suppose that the "top" string on a guitar and the "bottom" string on a guitar have the same tension. Which string has a greater linear density (mass/length)? Which string will have the greater fundamental frequency?
4. Sketch the first four harmonics for a standing wave on a string that is fixed at both ends. Label each graph with the harmonic (n). If the length of the string is 1 m, write the wavelength for each harmonic.