Physics 211, Fall 2008

Quiz 5, Form: A

Name: Key

Date:

$G = 6.7 \times 10^{-11}$ Nm^2/kg^2

point particle: $I = mr^2$

hollow wheel: $I = MR^2$

solid wheel: $I = \frac{1}{2}MR^2$

solid sphere: $I = \frac{2}{5}MR^2$

Section 1. Multiple Choice

1. Suppose that a test tube in a centrifuge in a biology lab rotates at a rate of 6000 rpm (i.e. revolutions per minute) in a radius of 0.20 m. What is the linear speed v of the test tube, in m/s?

   (a) 7540 m/s
   (b) 20.0 m/s
   (c) 628 m/s
   (d) 126 m/s
   (e) 1200 m/s

   $\omega = \left(6000 \frac{rev}{min}\right)\left(\frac{2\pi}{rev}\right)\left(\frac{1 min}{60 s}\right) = 200\pi \frac{rad}{s}$

   $v = \omega R = (200\pi)(0.2) = 126 \frac{m}{s}$

2. An ice skater is performing a spin (on ice of course). Initially, she is in a layback spin, where her free leg is held outward, her back is arched, and her arms are extended outward as shown below.

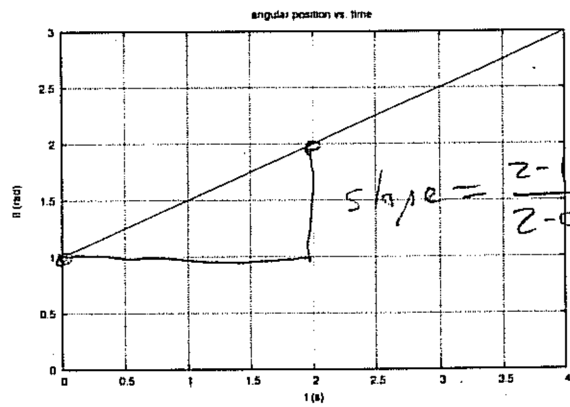
   If during the spin, she stands vertically and brings her arms and leg closer to her axis of rotation, her *angular momentum*

   (a) increases.
   (b) decreases.
   (c) remains constant.

3. As the skater in the previous question brings her arms and leg closer to her axis of rotation, her *moment of inertia*

   (a) increases.
   (b) decreases.
   (c) remains constant.

4. The angular position as a function of time for an object moving in a circle is shown below.



   What is the angular velocity of the object?

   (a) 0.5 rad/s
   (b) 0.75 rad/s
   (c) 1.0 rad/s
   (d) 1.5 rad/s
   (e) 2.0 rad/s

   $\omega = \frac{\Delta\theta}{\Delta t}$ = slope of θ vs. t graph; slope $= \frac{2-1}{2-0} = \frac{1}{2}$; $\omega = 0.5 \frac{rad}{s}$

5. Which has a greater moment of inertia, a *solid sphere* of mass M and radius R or a *hollow cylinder* of the same mass and radius, if each one rotates about its center of mass?

   (a) a solid sphere
   (b) hollow cylinder
   (c) neither, because they have the same moment of inertia

6. A satellite orbits Venus at a distance r. The gravitational force on the satellite due to Venus is F. If it is put into an orbit that is $0.8r$, then the gravitational force on the satellite will be

(a) the same, F.
(b) $0.64F$.
(c) $0.8F$.
(d) $1.25F$.
(e) $1.56F$.

$F_{grav} \propto \frac{1}{r^2}$

$\frac{1}{(0.8r)^2} = \frac{1}{0.64r^2}$

$\approx \frac{1.56}{r^2}$

Note: $\frac{1}{0.64} = 1.56$

7. You do an experiment where you use video analysis to measure x(t) for an object moving in circular motion. Fitting a curve to the function gives $x = 0.625\cos(7.14t + 0.010)$ where t is in seconds and x is in meters. What is the *radius* of the object's motion?

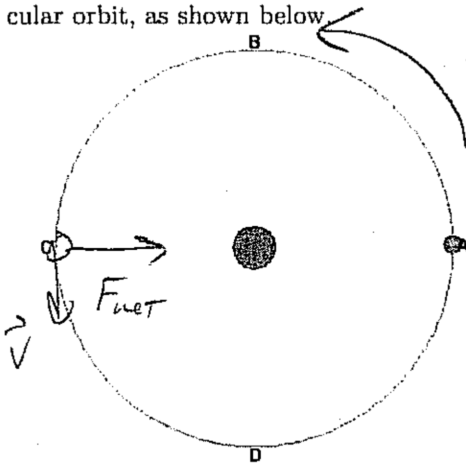
(a) 7.14 m
(b) 0.010 m
(c) 0.880 m
(d) 0.625 m
(e) 1.25 m

$x = R\cos\theta$

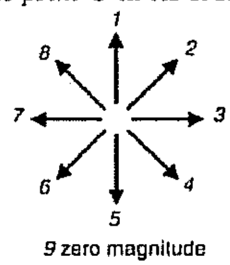
$x = R\cos(\omega t + \theta_0)$

$= 0.625\cos(7.14t + 0.01)$

8. A planet orbits a star *counterclockwise* in a circular orbit, as shown below.



Which arrow below points in the direction of the net force on the planet when the planet is at *point C* in its orbit?



net force is radial, toward the center

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

$\vec{F}_{net}$ at point C

9. Which arrow (in the previous question) points in the direction of the velocity of the planet when it is at *point C* in its orbit?

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

$\vec{v}$ at point C

10. A solid flywheel in an exercise bike has a radius 0.20 m and a mass of 10 kg. You start pedaling and accelerate the wheel at a rate of 2.0 rad/s^2. What is the net torque on the wheel? (Hint: use Newton's second law for rotation.)

(a) 0.80 N · m
(b) 0.40 N · m
(c) 20 N · m
(d) 4.0 N · m
(e) 8.0 N · m

$\tau_{net} = I\alpha = \left(\frac{1}{2}MR^2\right)\alpha = \frac{1}{2}(10)(0.2)^2(2) = 0.4$ N·m

11. A satellite of mass 1000 kg orbits Earth which has a mass of 6×10^{24} kg. On which object is the gravitational force (due to the other object) the greatest?

(a) The gravitational force on Earth due to the satellite.
(b) The gravitational force on the satellite due to Earth.
(c) Neither, because the gravitational force on each object is the same.

Newton's 3rd law

12. A child rolls a bowling ball down the lane. It rolls without slipping. If the ball has a mass of 3.63 kg and radius 21.6 cm and a center-of-mass speed of 1.0 m/s, what is its angular speed?

(a) 1.36 rad/s
(b) 0.216 rad/s
(c) 35.6 rad/s
(d) 1.0 rad/s
(e) 4.63 rad/s

$V_{c.m.} = R\omega$

$\omega = \frac{V_{c.m.}}{R} = \frac{1.0 \text{ m/s}}{0.216 \text{ m}}$

$\omega = 4.63$ rad/s

13. A wheel rotates *clockwise* at a constant rate of 2.5 rev/s. What is its *angular velocity* in rad/s? (Note: pay attention to the sign.)

(a) 15.7
(b) -2.5 rad/s
(c) -900 rad/s
(d) -15.7 rad/s
(e) 0.398 rad/s

$\omega = -2.5$ rev/s

$\left(-2.5 \frac{rev}{s}\right)\left(\frac{2\pi \text{ rad}}{rev}\right) = -15.7$ rad/s

14. If a wheel is rotating *clockwise* and is *speeding up*, then its angular acceleration is

(a) positive.
(b) negative.
(c) zero.

ω is −

speeding up so α and ω have the same sign

15. A gymnast is doing the high bar routine. At some point during the routine, he is moving in a circle around the bar. If his speed when is below the bar is 7.67 m/s, what is the tension in his arms? His mass is 65 kg, and the radius of his circular path is 1.5 m, measured from his hands to his center of mass.

(a) 2550 N
(b) 640 N
(c) 1910 N
(d) 3190 N
(e) 1280 N

$F_{net\ rad} = \frac{mv^2}{R}$

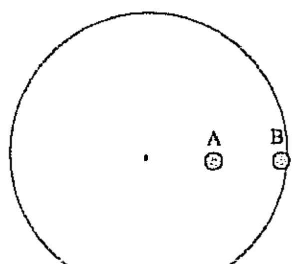
$T - mg = \frac{mv^2}{R}$

$T = \frac{mv^2}{R} + mg = 3186$ N

16. What is the condition necessary for the angular momentum of a system to be conserved?

(a) The net torque on the system must equal zero.
(b) The system must not be rotating.
(c) The angular velocity of each object in the system must remain constant.
(d) The objects within the system cannot exert forces or torques on each other.
(e) The moment of inertia of the system must remain constant.

17. Two children, A and B, ride on a merry-go-round that rotates at a constant speed, as shown below.



$F_{net} = \frac{mv^2}{R}$

$v = \omega R$ so

$F_{net} = \frac{m\omega^2 R^2}{R}$

On which child is the *net force* the greatest (magnitude)?

(a) A
(b) B
(c) Neither, because the net force on each child is the same.

greater R, greater net force needed for circular motion

18. Which child (in the previous question) has a greater *angular* speed?

   (a) A

   (b) B

   (c) Neither, because they have the same angular speed.

$\omega = \frac{2\pi}{T}$ they both make 1 revolution in the same time

19. If the merry-go-round (in the previous question) rotates at a rate of $\pi/2$ rad/s, what is its period?

   (a) $\pi/2$ s

   (b) 0.50 s

   (c) 2.0 s

   (d) 0.25 s

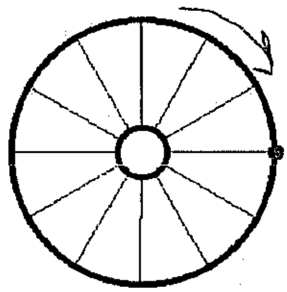
   (e) 4.0 s

$\omega = \frac{2\pi}{T} = \frac{\Delta\theta}{\Delta t}$

$T = \frac{2\pi}{\omega}$

$T = \frac{2\pi}{\frac{\pi}{2}} = 4\,s$

20. The wheel shown below initially rotates *clockwise* at a rate of 3 rev/s. I gently press the rim of the wheel with my hand, causing the wheel to slow down. If it slows down to 1 rev/s clockwise in 5.0 s, what is the angular acceleration of the wheel in rad/s^2? (Note: make sure the sign is correct.)



clockwise

$\omega_i = -3\ \frac{rev}{s}$

$\omega_f = -1\ \frac{rev}{s}$

$\Delta t = 5\,s$

   (a) 2.51 rad/s^2

   (b) -2.51 rad/s^2

   (c) 0.4 rad/s^2

   (d) -0.4 rad/s^2

   (e) -3.77 rad/s^2

$\alpha = \frac{\Delta\omega}{\Delta t}$

$= \frac{\omega_f - \omega_i}{\Delta t}$

$= \frac{-1-(-3)}{5}$

$= \frac{2}{5} = 0.4\ \frac{rev}{s^2}$

$= \left(0.4\ \frac{rev}{s^2}\right)\left(\frac{2\pi\ rad}{rev}\right)$

$= 2.51\ \frac{rad}{s^2}$

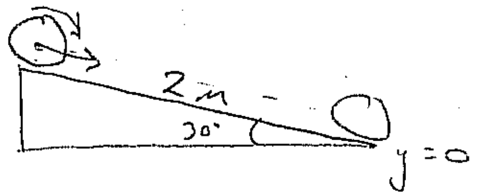
Section 2. Problem Solving

21. A solid wheel is released from rest at the top of a 2-m long incline making an angle of 30° with the horizontal. The wheel has a mass of 0.25 kg and a radius of 0.10 m. If the wheel rolls down the incline without slipping, what is its speed when it gets to the bottom?

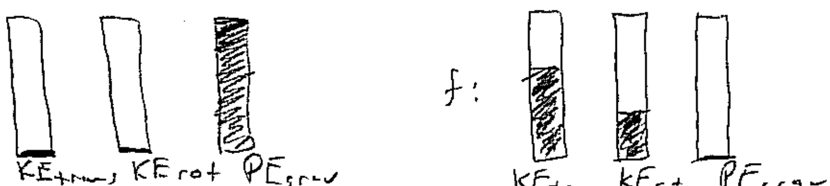
    (a) Sketch a picture of the situation.

$h = 2\sin(30) = 1\ m$

$y_i = h$



    (b) Sketch a bar graph showing the initial translational kinetic energy, initial rotational kinetic energy, and initial gravitational potential energy and a bar graph showing the final translational kinetic energy, final rotational kinetic energy, and final gravitational potential energy, like those drawn in class. Be sure that it is clear from your pictures that the total energy is constant.



Note: $KE_{trans_f} + KE_{rot_f} = PE_{grav_i}$; the proportion of KE_{trans} and KE_{rot} is not important

    (c) What will be the speed of the wheel at the bottom of the incline?

$$E_i = E_f$$

$$PE_{grav_i} = KE_{trans_f} + KE_{rot_f}$$

$$mgy_i = \frac{1}{2}mv_{cm_f}^2 + \frac{1}{2}I\omega_f^2 \qquad \omega_f = \frac{v_{cm_f}}{R}$$

$$mgy_i = \frac{1}{2}mv_{cm_f}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{cm_f}}{R}\right)^2$$

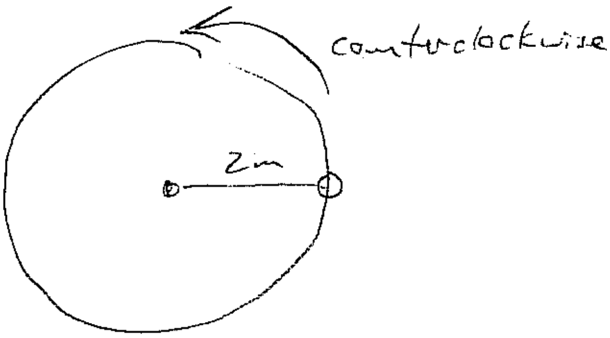
$$mgy_i = \frac{1}{2}mv_{cm_f}^2 + \frac{1}{4}mv_{cm_f}^2$$

$$mgy_i = \frac{3}{4}mv_{cm_f}^2$$

$$v_{cm} = \sqrt{\frac{4}{3}gy_i} = \sqrt{\frac{4}{3}(9.8)(1\,m)} = \boxed{3.61\ \frac{m}{s}}$$

22. A child is standing on a merry-go-round while it rotates counterclockwise with a constant speed of 0.80 rad/s. Initially she is standing 2.0 m from the center of the merry-go-round. She jumps off in such a way that she is moving radially outward when she hits the ground. The merry-go-round has a mass of 100 kg and a radius of 2.0 m. The child's mass is 30 kg. What is the final angular velocity of the merry-go-round after the child jumps off? (Hint: angular momentum is conserved.)

i:



$\omega_i = 0.8 \frac{rad}{s}$

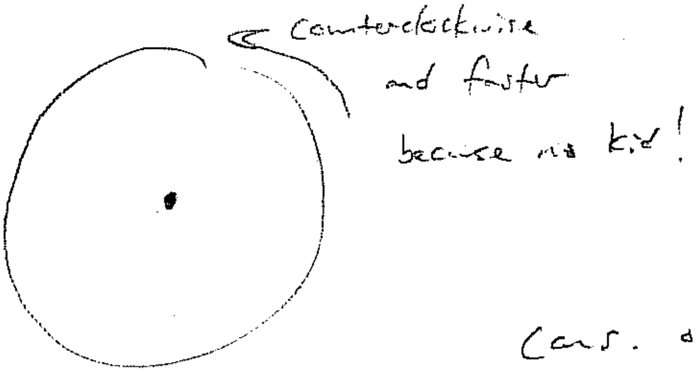
$r_{i\,kid} = 2m$, $M_{kid} = 30\ kg$

$M_{mgr} = 100\ kg$

$R_{mgr} = 2m$

$\omega_f = ?$

f:



cons. of angular momentum

$L_i = L_f$

$I_i \omega_i = I_f \omega_f$

$\omega_f = \frac{I_i}{I_f} \omega_i$

$\omega_f = \frac{320}{200} \omega_i$

$= \left(\frac{320}{200}\right)(0.8)$

$= \boxed{1.28 \frac{rad}{s}}$

$I_i = I_{m.g.r} + I_{kid}$

$= \frac{1}{2}MR^2 + M_{kid} r_{kid}^2$

$= \frac{1}{2}(100)(2)^2 + 30(2)^2$

$= 200 + 120$

$= 320\ kg\,m^2$

$I_f = \frac{1}{2}MR^2 = 200\ kg\,m^2$