# Chapter 9

## Terms

Be able to define or discuss the following terms and ideas, with their SI units if appropriate.

1. center of mass
2. total momentum of a multi-particle system
3. relative position (relative to the center of mass)
4. relative velocity (relative to the center of mass)
5. total kinetic energy
6. translational kinetic energy
7. relative kinetic energy
8. rotational kinetic energy
9. momentum of inertia

## Equations

Understand the meaning and know the SI units of all symbols in these equations; know how to perform each mathematical operation, such as trig functions; know how to solve for any unknown quantity; understand how changing one quantity affects another quantity (if all other quantities remain constant); be able to apply one or more equations to solve a problem.

- Center of mass:

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

- Velocity relative to the center of mass

$$\vec{v}_{rel} \quad = \quad \vec{v} - \vec{v}_{cm}$$

- Total momentum of a system of particles:

$$\vec{P}_{tot} = M\vec{v}_{cm} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots$$

- Total kinetic energy of a system of particles (non-relativistic):

$$K_{tot} = \sum_{i=1}^{N} \frac{1}{2} m v_i^2$$

- Translational kinetic energy:

$$K_{trans} = \frac{1}{2} M v_{cm}^2$$

- Total relative kinetic energy (relative to the center of mass):

$$K_{rel} = \sum_{i=1}^{N} \frac{1}{2} m_i v_{i_{rel}}^2$$

- Total kinetic energy of a system of particles (non-relativistic):

$$K_{tot} = K_{trans} + K_{rel} = \frac{1}{2} M v_{cm}^2 + \sum_{i=1}^{N} \frac{1}{2} m_i v_{i_{rel}}^2$$

- Conservation of Energy for the point-particle system:

$$\Delta K_{trans} = \vec{F}_{net} \cdot \Delta \vec{r}_{cm} \quad \text{for a constant net force}$$

- Conservation of Energy for the real system:

$$\begin{aligned} \Delta E_{sys} &= W + Q \\ \Delta E_{sys} &= \vec{F}_1 \cdot \vec{r}_1 + \vec{F}_2 \cdot \vec{r}_2 + \cdots + Q \end{aligned}$$

- Rotational kinetic energy:

$$K_{rot} = \frac{1}{2} I \omega^2$$

- Moment of inertia:

$$\begin{aligned} \text{point particle:} \quad I &= mr^2 \\ \text{solid disk rotating about its center:} \quad I &= 1/2MR^2 \\ \text{hollow ring rotating about its center:} \quad I &= MR^2 \\ \text{sphere rotating about its center:} \quad I &= \frac{2}{5} MR^2 \end{aligned}$$

## Skills

1. Apply Conservation of Energy to a system whose center of mass motion is different than the motion of the particles in the system.
2. Calculate relative velocity for each particle in a system of particles.

## Lab Skills

1. Use video analysis to measure the center of mass motion of a system and the relative motion of a particle in the system.