Video Analysis of a Bicycle Wheel in Uniform Circular Motion

Apparatus

Tracker software (free; download from http://www.cabrillo.edu/∼dbrown/tracker/)
video: `bicycle-wheel.mov` from http://physics.highpoint.edu/∼atitus/videos/

Goal

In this experiment, you will measure and graph the x-position, y-position, and angle as a function of time for a sticker on a bicycle wheel rotating in a circle at constant speed. You will use the data to determine the sticker's linear speed, angular speed, period, and radius.

Introduction

Suppose an object moves counterclockwise along a circular path. Figure 1 shows an object at intervals of 1/30 s between the first image A and the last image G.

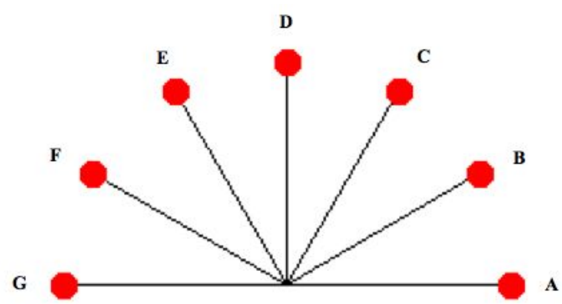

Figure 1: An object moves in a circular path.

Suppose that we define $t = 0$ to occur at the first position of the ball. On the picture, label the time t for each subsequent position of the ball.

If you were to graph the x-position of the object as a function of time, what do you think the graph would look like? (No numbers are needed, just a qualitative sketch.)

The x-position of the object for each image is shown in Figure 2.

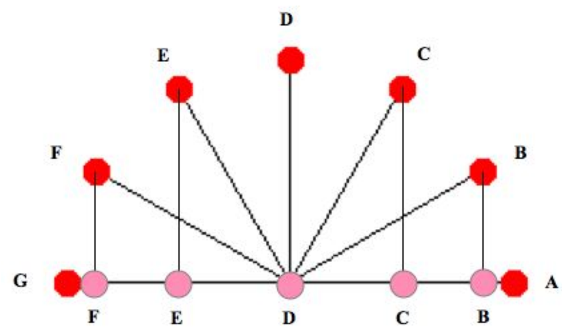

Figure 2: The x-position of an object moving in a circular path.

If you just look at the x-position of the object from A to G, what does its motion remind you of?

The radius of the circle is R. We can calculate the object's x and y position at any instant. Let's look at the object when it is at position C. The triangle showing the object, its x-position, and its y-position is shown below.

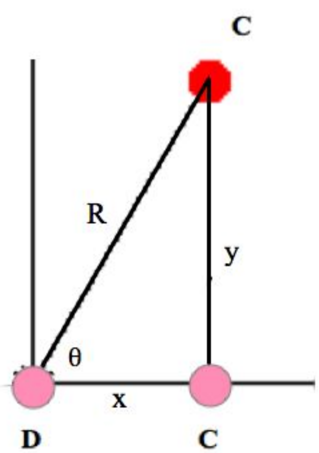

Figure 3: The x and y components of the object's position.

Using this angle, the x-position and y-position can be calculated as

$$x = R\cos(\theta)$$
$$y = R\sin(\theta)$$

For an object moving in circular motion with a constant speed, the angle θ that the object makes with the +x axis changes at a constant rate. The rate that the angle changes is called the *angular speed*. To calculate angular speed, you measure how much it turns ($\Delta\theta$) and divide by the time interval. Thus,

$$\omega = \frac{\Delta\theta}{\Delta t}$$

If θ is the angle at any instant t and if θ_0 is the initial angle at $t = 0$, then

$$\omega = \frac{\theta - \theta_0}{t}$$

$$\theta = \omega t + \theta_0$$

Thus, the x-position and y-position of the object at the clock reading t is

$$x = R\cos(\omega t + \theta_0)$$
$$y = R\sin(\omega t + \theta_0)$$

As a result, the x-motion and y-motion each resemble simple harmonic motion.

The *linear speed* of the object is distance traveled per second. It's easiest to consider one complete rotation. The distance traveled around a circle is the *circumference*. The time for one revolution is the *period*. Thus, the linear speed of the object is

$$|\vec{v}| = \frac{2\pi R}{T}$$

The angular speed during one revolution is $\omega = 2\pi/T$. Therefore, we can write the linear speed as

$$v = \omega R$$

where ω is in units of rad/s. It is typical to drop the magnitude symbol and vector symbol and write the speed $|\vec{v}|$ more simply as v.

In Figure 1, the time interval between positions is 1/30 s, and the angle it turns, between successive positions, is 30°. What the object's angular speed in degrees per second?

Often, we use units of radians instead of degrees when measuring angles. 180° is π radians. What is the angular speed of the object in Figure 1 in radians per second?

Procedure

1. Download the video `bicycle-wheel.mov`. This video was recorded at 300 fps, though it plays back at 30 fps and appears in slow motion.
2. Open *Tracker* and import the video.
3. Open the Clip Settings window by clicking on the **Clip Settings** icon (shown in Figure 4) which is part of the video control toolbar that is below the video clip.

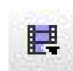

Figure 4: The Clip Settings icon.

4. In the **Clip Settings** pop-up window, enter a frame rate of 300 fps, as shown in Figure 5

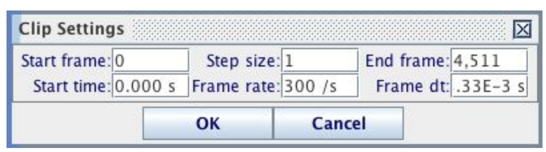

Figure 5: The Clip Settings icon.

5. Go to the first frame of the video and use the meterstick to calibrate distance in the video.
6. Advance to the next frame (frame number 001).
7. Place the origin at the axle (at the center of the white disk at the hub of the wheel in the video).
8. Since there are 300 frames of video recorded per second, the time interval between frames is quite small. As a result, we can skip frames between marking the ball and thus take fewer data points. Click on the **Step Size** button, as shown in Figure 6 and change it to **5**.

Figure 6: Change the step size in order to skip frames.

9. Click the Create button in the toolbar and create a new point mass.

10. Make sure the video is on the first frame.

11. Hold the shift key down and click once on the green sticker that is on the bicycle tire. The video will then advance 5 frames. Continue to mark the green sticker for a total of 4 revolutions. If you are only displaying the last few marks, then it will look something like Figure 7.

Figure 7: Marks for the green sticker on the rotating bicycle wheel.

Analysis

1. Right-click on the x vs. t graph and select **Analyze** .

2. Check the **Fit** checkbox.

3. In the curve fit pane, Click the Fit Builder... , click the button Clone , and select "Sinusoid." Click Add to add a parameter d with value 0. Edit the Sinusoid1 function to be $a * \cos(b * t + c) + d$, as shown in 8. The additive constant d will shift the curve fit up and down by a constant as needed, though it should be 0.

4. The Autofit algorithm does not do well with sinusoidal curves. Thus, you will need to adjust each parameter manually. When you get these values fairly close, then you can do an Autofit. It helps to start with some reasonable values. So make some approximate guesses for the following functions based on the curve:

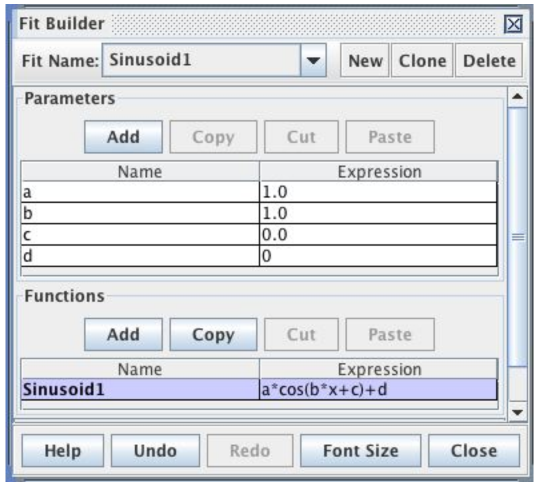

Figure 8: The Fit Builder is used to fit a curve to a user-defined function.

d shifts the curve up or down. It should be zero (or close to zero).

a is the maximum value on the curve. What is approximately the maximum value of a? Enter this into the box for a.

b is the angular velocity, $2\pi/T$. What is the approximate period of the oscillation? Use it to calculate an approximate value of b, and enter this value into the box.

c is the phase. Once you set a and b, it will be easy to determine c. It shifts the curve right or left. You can adjust this manually without knowing the initial value.

If you begin with approximate values, you can click in the Parameter Values box and click the up and down arrows to make small adjustments. You can also change the step size as you hone in on the best value for the parameter. Adjust each parameter by very small amounts (steps of 0.1%) until you get the best curve-fit possible.

An example curve-fit is shown in Figure 9.

What are the curve parameters and the fit equation for the best-fit curve?

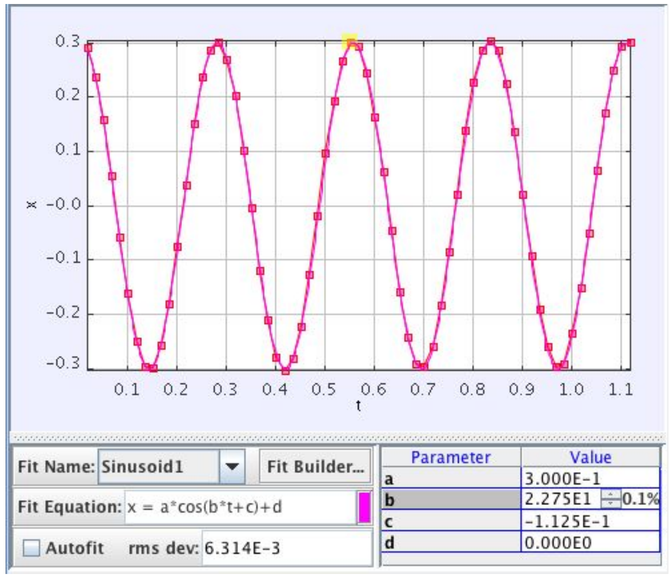

Figure 9: The best-fit curve for a point on the wheel.

From the curve fit, determine the radius of the sticker R.

How does your result compare to the measured radius of the sticker which is 30 cm?

From the curve fit, determine the angular velocity of the wheel ω.

From the curve fit, determine the initial angle of the green sticker θ_0.

What is the period T of the wheel?

What is the linear speed v of the green sticker?

5. Close the Data Tool window and return to the main Tracker window. Change the graph to plot y vs. t. Right-click the graph and select **Analyze** . Repeat the same procedure as before, but this time fit a curve to $y = a * \sin(b * t + c) + d$.

Record the values for the curve fit and compare to what you found when analyzing $x(t)$.

6. Close the Data Tool window and return to the main Tracker window. Change the graph to plot θ vs. t. Right-click the graph and select **Analyze** . The angle θ is calculated using

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

As a result, it is defined between $-\pi$ and π. Whenever the angle exceeds π, it is then given as a negative angle. This is standard practice for computers and calculators. As a result, your graph of θ vs. t will look like Figure 10.

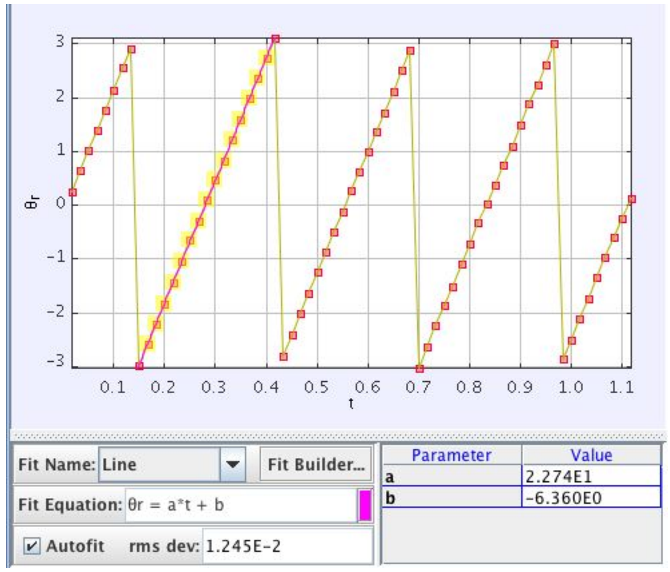

Figure 10: θ vs. t.

For each linear part of the graph, do a linear curve fit as shown in Figure 10. From this curve fit, record the angular velocity ω. Record the angular velocity for each revolution or part of a revolution.

$\omega_1 =$

$\omega_2 =$

$\omega_3 =$

$\omega_4 =$

$\omega_5 =$

What do you notice about your measurement of ω for each revolution? How can you explain this observation?

Application

(a) If the wheel is instead rotating clockwise, how would it affect the graph of θ vs. t? How would it change your measurement of ω from the θ vs. t graph?

(b) In the video, you will notice a white sticker on a spoke that is at a radius of a little less than half the radius of the green sticker. If you measure the motion of this white sticker, which of these variables will be the same as that of the green sticker? (R, ω, θ_0, v, T)