

Conservation of Angular Momentum of a Spinning Figure Skater

Apparatus

Tracker software (free; download from <http://www.cabrillo.edu/~dbrown/tracker/>)
video: `figure-skater.mov` from <http://physics.highpoint.edu/~atitus/videos/>

Goal

In this experiment, you will measure the initial and final angular velocity of a figure skater doing a “scratch spin” and you will calculate her change in moment of inertia.

Introduction

The “scratch spin” is a classic figure skating maneuver in which the skater draws her arms and leg in, which causes her angular speed to increase. This is the result of conservation of angular momentum. As the skater reduces her moment of inertia by pulling her arms and legs in, closer to the axis of rotation, her angular speed increases in order to maintain constant angular momentum.



Figure 1: A skater executing a scratch spin.

Newton’ second law for rotation (sometimes referred to as The Angular Momentum Principle) is

$$\tau_{net} = \frac{d\vec{L}}{dt}$$

where τ_{net} is the net torque on the system about some point P and \vec{L} is the angular momentum of the system about the same point. If the system is a rigid body rotating about an axis, then

$$\vec{L} = I\omega\vec{e}_a$$

where I is the moment of inertia of the body about the rotation axis and $\vec{\omega}$ is the angular velocity about the same axis.

If the net torque on a system is zero, then its angular momentum is constant. This principle is called **Conservation of Angular Momentum**. For a figure skater spinning on the ice, the net torque on her is nearly zero, and her angular momentum is constant, and

$$\vec{L}_i = \vec{L}_f$$

If we define her rotation axis to be the z -axis, then

$$I_i\omega_i = I_f\omega_f$$

If she is spinning and she pulls her arms and leg inward, then her moment of inertia decreases. Since $I\omega$ is constant, then this results in an increase in her angular velocity ω and she rotates faster.

If she is spinning and she brings her arms and leg outward, then her moment of inertia increases. Since $I\omega$ is constant, then this results in a decrease in her angular velocity ω and she rotates slower.

Procedure

The following instructions assume that you are familiar with using Tracker.

1. Import the video *figure-skater.mov* into Tracker.
2. Use the skater's height to calibrate distance in the video. Her height from the bottom of her skates to the top of her head is 1.7 m.
3. Set the frame step size to 5 in order to skip 4 frames between data points. You will still have plenty of data to analyze her motion.

Note that this is still a lot of data. If you want to further reduce your data set, you can increase the step size to 10, though you'll sacrifice some precision and accuracy perhaps.

4. Create a point mass and mark the skater's left hand. It is best to mark the base of her hand, closer to her wrist. When her hand disappears behind her body, do not skip the frame. Rather, guess at where the hand might be and mark it there.
5. Look at the graph of $x(t)$. You'll notice the oscillatory motion that is indicative of uniform circular motion; however, the data drifts "downward" on the graph in the $-x$ direction. What I mean is that it doesn't oscillate about a constant x value.

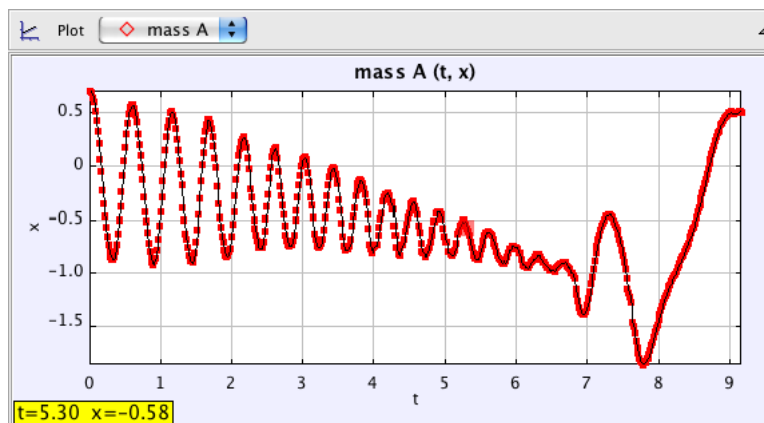


Figure 2: A graph of $x(t)$ for the skater's left hand.

Why does the data drift in the $-x$ direction?

So that we can measure her rotation relative to her center of mass, we have to mark her center of mass. Make the approximation that her center of mass is at the center of her waist.

6. Create a new point mass and name it *center of mass*. Mark the center of her waist in all of the frames.
7. Look at the graph of $x(t)$ for her center of mass. In what direction is she moving?

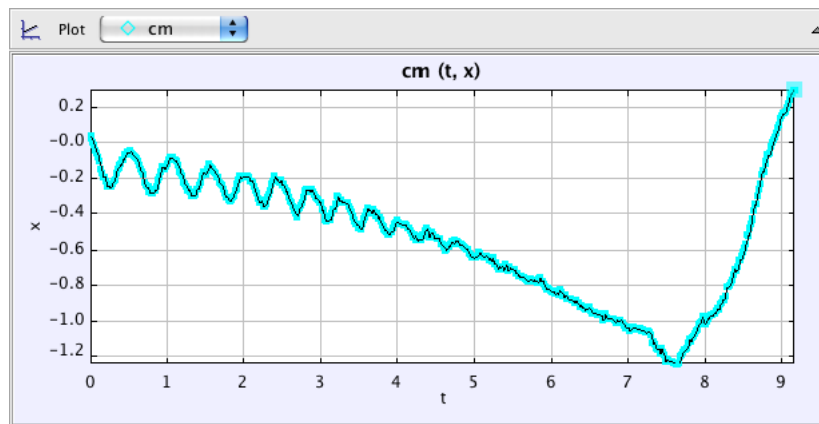


Figure 3: A graph of $x(t)$ for the skater's center of mass.

We would like to analyze her rotational motion (relative to her center of mass). Thus, we must subtract off her center-of-mass motion. Tracker makes this very easy for us.

8. Go to the menu **Coordinate System**→**Reference Frame** and select “center of mass” in order to measure the motion of all objects relative to the center of mass.
9. Change the graph to show $x(t)$ for her left hand.

Does her hand now rotate around $x = 0$?

Analysis

1. Right-click the graph of $x(t)$ for the skater's left hand and select **Analyze...**

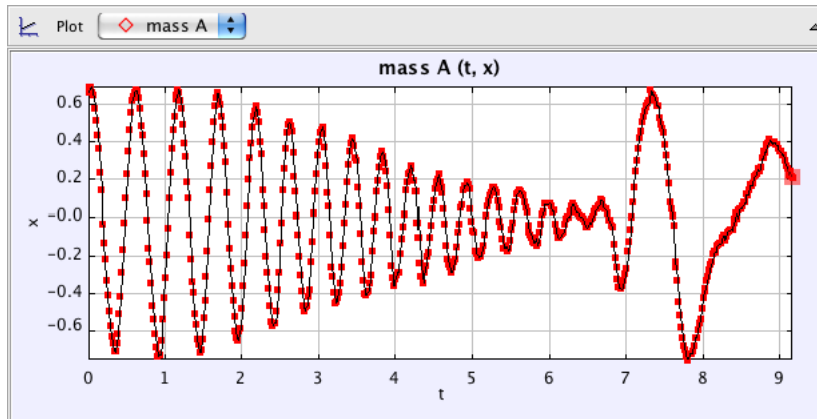


Figure 4: A graph of $x(t)$ for the skater's left hand relative to her center of mass.

- In the Data Tool, highlight one cycle of data when her arms are fully extended. Fit a sinusoidal curve to the data.

What is the resulting curve fit? From this equation, determine the radius of the circle traveled by her hand and her angular velocity ω

- Highlight one cycle of data when her arms are over her head. Fit a sinusoidal curve to the data.

What is the resulting curve fit? From this equation, determine the radius of the circle traveled by her hand and her angular velocity ω

- What is the ratio $\frac{\omega_f}{\omega_i}$?
- What is the ratio $\frac{I_f}{I_i}$?