Waves

Apparatus

Tracker software (free; download from http://www.cabrillo.edu/∼dbrown/tracker/)
Tracker Digital Library: http://physics.highpoint.edu/∼atitus/tracker/standing-waves/

Goal

Using video analysis, you will measure the frequency and wavelength of standing waves on a "string" (actually a long spring) and will calculate the speed of the wave. You will investigate which properties of the string affect the wave speed.

Introduction

A standing wave exists when two waves of the same speed travel in opposite directions and interfere. If the wavelength of the wave is such that there is a node at the beginning and end of the string. The longest wavelength for this to occur is when $\lambda = 2L$ which is shown in Figure 1.

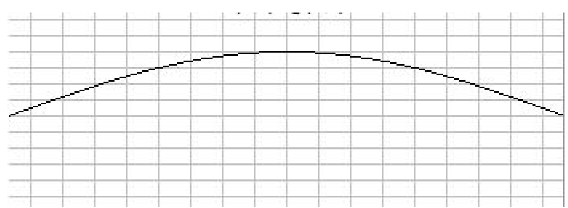

Figure 1: First harmonic, n=1 (the fundamental).

If the wavelength is shorter, the next possible standing wave is for $\lambda = L$ which is shown in Figure 2.

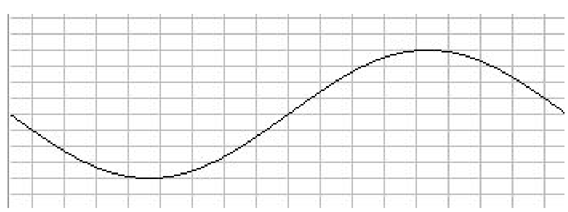

Figure 2: Second harmonic, n=2.

If the wavelength is shorter by half a wavelength, the next possible standing wave is for $\lambda = \frac{2}{3}L$ which is shown in Figure 3.

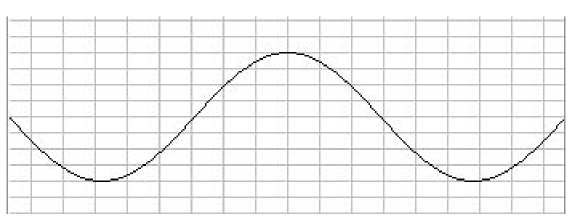

Figure 3: Third harmonic, n=3.

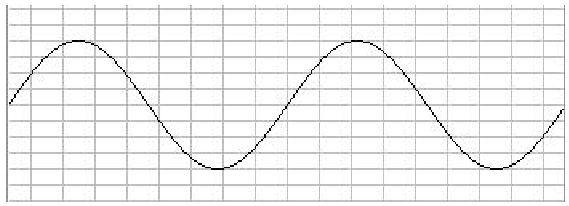

Figure 4: Fourth harmonic, n=4.

Again, shorten the wavelength by half a wavelength, and you'll get $\lambda = \frac{1}{2}L$ which is shown in Figure 4. We can write a general equation relating wavelength of the standing wave to the length L of the string.

$$\lambda = \frac{2L}{n} \tag{1}$$

where $n = 1, 2, 3, 4, ...$ and each integer is referred to as the *mode* or *harmonic*. The longest wavelength occurs for $n = 1$, the *first harmonic* which is called the *fundamental*. Higher harmonics are referred to as *overtones*. The second harmonic is called the *first overtone*. You will notice that for a string that is fixed at both ends, the number of antinodes is the same as the harmonic n.

Speed of a wave on a string

For any wave, its speed is related to its wavelength and frequency.

$$v = \lambda f \tag{2}$$

The wave speed also depends on the medium it travels in. For a wave on a string, its speed depends on the tension in the string and the linear density of the string μ. (Linear density is its mass per unit length, so $\mu = m/L$). The speed of a wave on a string is

$$v = \sqrt{\frac{F_T}{\mu}} \tag{3}$$

Stringed Instruments

Equating the above two equations, then

$$\lambda f = \sqrt{\frac{F_T}{\mu}} \tag{4}$$

For a standing wave on a string, substitute $\lambda = \frac{2L}{n}$ and solve for the frequency of the standing wave in terms of tension and linear density of the string, as well as the harmonic n.

$$\left(\frac{2L}{n}\right) f = \sqrt{\frac{F_T}{\mu}} \tag{5}$$

$$f = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}} \tag{6}$$

Suppose that a standing wave on a string on an instrument (like piano or violin or guitar, etc.) vibrates with the fundamental frequency ($n = 1$). Then,

$$f_1 = \frac{1}{2L}\sqrt{\frac{F_T}{\mu}} \tag{7}$$

To *increase* the frequency of the fundamental standing wave, you can:

- decrease the length of the string (by pressing down on the string).
- increase the tension (by tightening a screw, or "key").
- decrease the mass per unit length (by using a less dense string, typically a thinner string or one made of a different material)

What can you do to decrease the fundamental frequency on a string?

The frequencies of a standing wave are proportional to n. Thus, the frequencies of overtones (i.e. $n > 1$ are integer multiples of the fundamental f_1.

$$f_n = nf_1 \tag{8}$$

As a result, the second harmonic has a frequency $2f_1$, the 3rd harmonic is $3f_1$, etc.

Experiment–Video Analysis of a Point on a Standing Wave

Procedure

1. Open Tracker.
2. Go to **File→Open Library Browser** .
3. Enter the URL `http://physics.highpoint.edu/~atitus/tracker/standing-waves/`
4. Double-click *n=1 lower tension* to open the video in Tracker.
5. Play the video and observe the motion of a piece of tape that is on the spring.

Describe the motion of the piece of tape in words. Based on what you learned in CH 21, what is this type of motion called?

If you measure and graph the vertical position of a piece of tape as a function of time, what should the graph look like if the tape's motion is simple harmonic motion? What does this remind you of?

Now, we will measure and graph $y(t)$.

6. To calibrate the video, you will use the 2-m long meterstick on the tray of the whiteboard in the video. In the toolbar, click on the **Calibration** icon shown in Figure 5.

Figure 5: Icon used to set the scale.

Choose the **Calibration Stick** option. Click and drag the left end of the calibration stick so that it is on top of the left end of the 2-m long stick on the tray of the whiteboard. Repeat for the right end of the calibration stick. Click on the number of the calibration stick to set its length to 2 m, as shown in Figure 6.

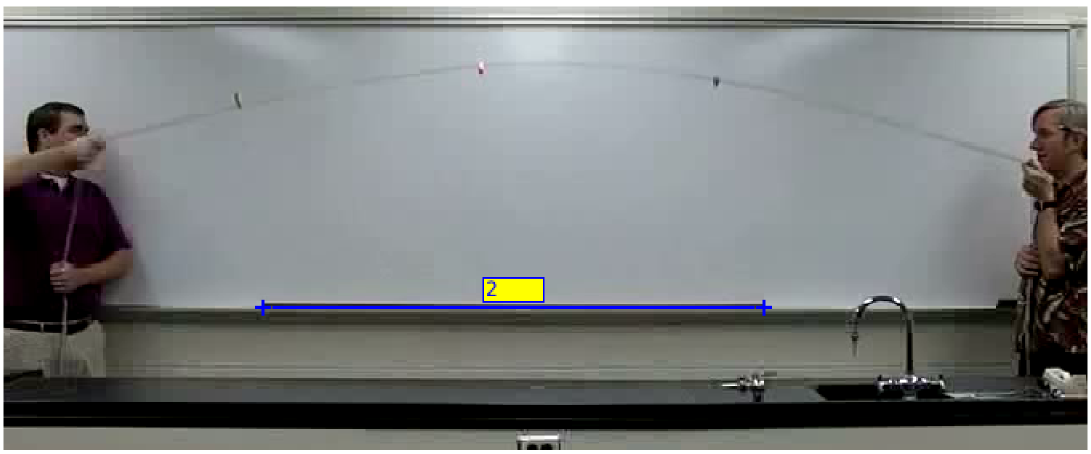

Figure 6: Set the scale of the calibration stick.

7. Click the **clip settings** icon  in the toolbar and set the **step size** to 5 frames.

8. To mark an object in the video, first click the Create button. From the menu, select **Point Mass** as shown in Figure 7. Now, you will mark the position of the object in each frame. **While holding down the shift key, click on the piece of tape that is on the spring, close to the center of the spring.** (There are actually three pieces of tape on the spring, but we'll just mark one of them for now.) The video will automatically advance to the next frame after you mark the object. Continue to mark the object for approximately 4 or 5 cycles of the spring. This might be about 90 data points.

9. We want to analyze the y-position of the marker as a function of time. To change the variable being graphed, click on the vertical axis variable on the graph and select **y: position y-component** as shown in Figure 8.

10. Right-click on the graph and select **Analyze...** . A new window with the graph will pop up. Click the Measure button and select **Coordinates** . Use this tool to measure time for any data point. Measure the total time between 4 peaks or 4 troughs and divide by the number of cycles to get the average period. Note that the time between four peaks corresponds to 3 cycles. Record your measurements in Table 1.

11. Observe the other two pieces of tape. (You do not have to measure their motion.)

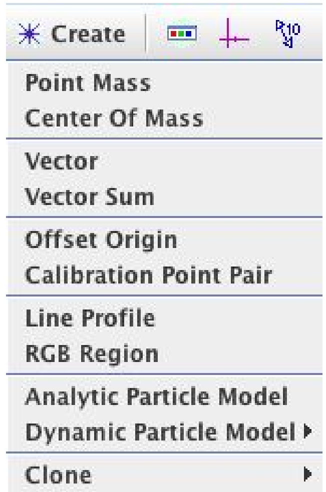

Figure 7: Create menu.

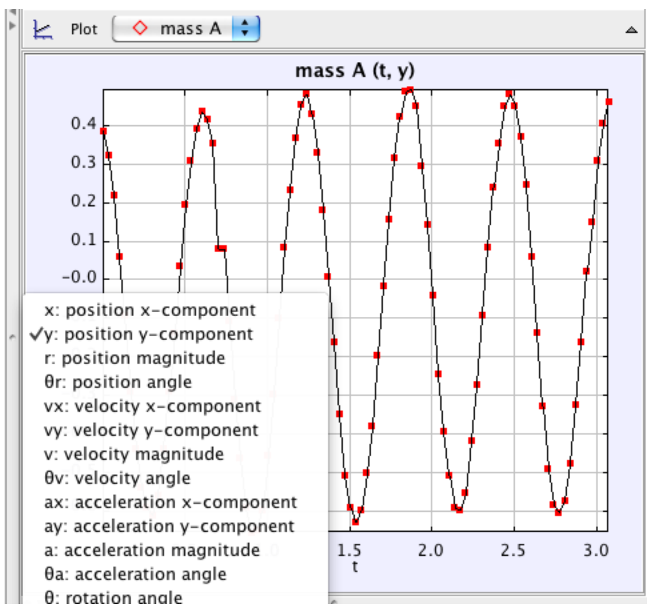

Figure 8: Change the variable plotted on the graph.

Do the three pieces of tape have the same periods?

Do the three pieces of tape have the same amplitudes?

12. Now you will measure the length of the spring. Go back to the main window. Click the Create button and select **Measuring Tools→Tape Measure** . Place the left end of the tape measure on the left person's hand. Place the right end of the tape measure on the right person's hand, as shown in Figure 9. Record the length of the spring in Table 1.

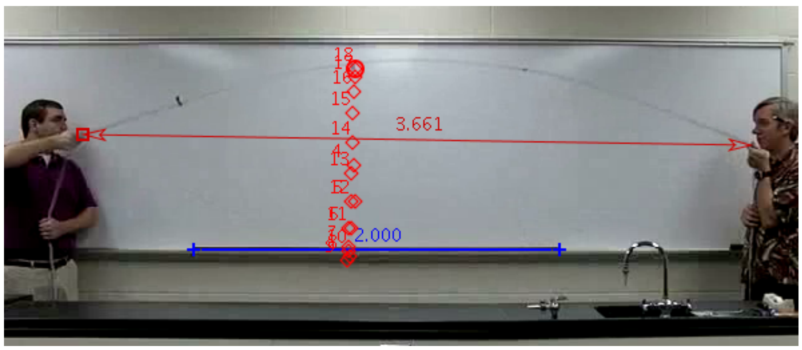

Figure 9: Measure the length of the spring.

Answer the following questions using your data for the period of the standing wave and the distance between the ends.

1. What is the frequency of the standing wave?
2. What is the wavelength of the standing wave?
3. What is the speed of the wave?

Record the results in Table 1.

13. Save the trk file to your computer in case you need it again. It will remain open as a tab in Tracker. When you open other files, they will be opened in new tabs. You can always close a tab if you don't need that Tracker file anymore.

14. Go to **File→Open Library Browser** and open the second video, *n=2 lower tension*. This standing

wave is not ideal because the amplitude for any particular point is not consistent. But the average period can still be measured. Repeat the previous steps to determine the period, frequency, length, wavelength and speed of the wave. **Note that it might be hard to see the piece of tape in all frames. As long as you click on the spring when marking the object, this is ok since we only care about the y-motion and since all pieces of the spring have the same period.**

15. Use a data table like the one below to record all of your data.

Table 1: Data

video	cycles	Δt (s)	T (s)	f (Hz)	L (m)	λ (m)	v (m/s)
n=1, lower tension							
n=2, lower tension							
n=3, lower tension							
n=1, higher tension							
n=2, higher tension							

Analysis

1. For each standing wave, calculate the speed v of the wave.

2. Is the speed of the wave approximately the same for any of the videos? If so, which ones? Calculate the average speed of the wave for those videos that have approximately the same speed.

3. Does the speed of the wave depend on the wavelength or frequency? Explain your reasoning.

4. What does the speed of the wave depend on? Explain your reasoning.

5. Suppose that the two people shorten the spring but stretch it to the same distance L as before. Does this result in greater tension or less tension? Greater linear density or less linear density?

6. You will notice that for the last two videos, the spring was stretched a greater amount. Stretching the spring increases the tension and decreases the linear density of the spring. Calculate the ratio of the average wavespeeds for the high tension and low tension cases.

$$\frac{v_{high\ tension}}{v_{low\ tension}} =$$

By what factor did the ratio T/μ increase when the spring was stretched?

$$\frac{(T/\mu)_{high\ tension}}{(T/\mu)_{low\ tension}} =$$

7. Suppose that the spring tension is kept constant. If you wiggle your hand faster, how does this effect the speed of a wave on the spring?

8. Suppose that the spring tension is kept constant. If you wiggle your hand faster until you find the next higher mode of standing wave, how does this effect the frequency of a standing wave on the spring? What about the wavelength?
9. What do you notice about the frequency for $n = 2$ compared to $n = 1$ (for a spring with the same tension and density)?
10. What do you notice about the frequency for $n = 3$ compared to $n = 1$ (for a spring with the same tension and density)?
11. Make a prediction: what would be the frequency and wavelength of a standing wave with $n = 4$ if the tension and density remains the same?
12. On a guitar string (or piano string or violin string), the spring approximately oscillates as a standing wave with $n = 1$ (i.e. the fundamental). So, this stays constant, regardless of other variables. If you increase the tension of the string:
    (a) Does the wavelength increase or decrease?
    (b) Does the frequency increase or decrease?
    (c) Does the speed of a wave on the string increase or decrease?
13. Suppose that the "top" string on a guitar and the "bottom" string on a guitar have the same tension.
    (a) Which string has a greater linear density (mass/length)?
    (b) Which string has the greater fundamental frequency?
14. Sketch the first five harmonics for a standing wave on a string that is fixed at both ends. Label each graph with the harmonic (n). If the length of the string is 0.5 m, what is the wavelength for each harmonic.